

Josephson junction array resonators in the Mesoscopic regime: Design, Characterization and Application

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by

Phani Raja Muppalla

Under the supervision of Univ-Prof. Dr. Gerhard Kirchmair, Department of experimental Physics, Technikerstaße 21A, Innsbruck-6020, Austria

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Abstract

This dissertation focuses on **Superconducting circuits**, a promising candidate for building a scalable quantum computer. An important architecture employed in the field is called Circuit Quantum Electrodynamics (circuit QED), where superconducting qubits are combined with high quality microwave cavities to study the interaction between artificial atoms and single microwave photons. The work on circuit QED performed in this thesis consist of two topics divided into three main projects:

1) Proposing an spin - spin interacting system using 3D transmon qubits in a rectangular cavity.

2) Characterizing a mesoscopic Josephson junction array resonator.

A 3D transmon qubit has a naturally occurring dipole moment. In **project 1**, from numerical simulations I investigate the interaction between two superconducting transmon qubits in a rectangular cavity. From this simulations, and along with theory collaborators, a novel platform for quantum many body simulations is proposed.

Josephson junction arrays have been investigated extensively since the 1980's, and have proven to be a promising highly non-linear building block for superconducting quantum circuits, ranging from qubits to parametric amplifiers, converters, quantum hybrid systems, high characteristic impedance circuits, etc. In **project 2**, a device based on 1000 JJA's has been characterized. As the device is in a so far unexplored regime where the anharmonicity is on the order of the linewidth, the bistability appears for a pump strength of only a few photons. The random switching between the two stable solutions around the bi-stable region is investigated by performing continuous time measurement.

The interplay between the non-linearity of the Josephson junction array and the coupling, provides a new resource for quantum non-demolition measurements. In **project 3**, an array of 18 Josephson junctions coupled to a superconducting qubit has been engineered, fabricated and characterized. The modified version named **JJAR.2.0** is designed to maximize measurements speed, achieve single-shot QND readout of qubit state.

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List of publications

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- [2] M. Dalmonte, S. I. Mirzaei, Muppalla, P. R., D. Marcos, P. Zoller, and G. Kirchmair. Realizing dipolar spin models with arrays of superconducting qubits. *Phys. Rev. B*, 92:174507, Nov 2015.
- [3] Muppalla, P. R., O. Gargiulo, S. I. Mirzaei, B. Prasanna Venkatesh, M. L. Juan, L. Grünhaupt, I. M. Pop, and G. Kirchmair. Bistability in a mesoscopic josephson junction array resonator. *Phys. Rev. B*, 97:024518, Jan 2018.

Abbreviations

FEM	Finite element method
JJAR	Josephson junction array resonator
JJAR.2.0	Modified Josephson junction array resonator
JBA	Josephson bifurcation amplifier
JPC	Josephson parametric converter
DJJPA	Dimmer Josephson junction parametric amplifier
JPA	Josephson parametric amplifier
\mathbf{Q}	Quality factor
MSR	Micro Stripline resonator
PEC	Perfect electrical conductivity
OFHC	Oxygen free copper
TEM	Transverse electric magnetic
TE	Transverse electric
\mathbf{TM}	Transverse magnetic
ϕ	Phase difference
R_N	Normal state resistance
I_c	Critical current of Josephson junction
E_J	Josephson energy
E_C	Charging energy
Q_{int}	Internal quality factor
Q_{tot}	Total quality factor
Q_{ext}	Coupling quality factor
κ	Line width
α	Anharmonicity

Chapter 1

Introduction

Quantum computation promises to solve certain problems more efficiently compared to a classical computer [1]. In quantum information processing, the classical bit with possible states 0 and 1 is replaced by the quantum bit or qubit that can assume any superposition state $|\psi\rangle = \alpha |g\rangle + \beta |e\rangle$, with qubit eigenstates $|g\rangle$ and $|e\rangle$. Due to the fundamental principle of quantum entanglement, the quantum state of N interacting qubits must be described by a common state in their joint Hilbert space of 2^N dimensions and in general cannot be decomposed into a product state of N single qubit states. When solving a quantum problem on a classical hardware, the computer needs to keep track of all probability amplitudes for any possible configuration of the system at any time, leading to an exponential increase in the computational power and memory requirement.

A prominent example for an exponential speed-up of quantum computers is prime factorization based on Shor's algorithm [2]. This is known as a hard problem for classical computers. Several proof-of-principle implementations of a compiled version of Shor's algorithm with a pre-defined small number have been demonstrated in nuclear magnetic resonance [3], with cold atoms [4], on a Photonic chip [5], with trapped ions [6], and with superconducting circuits [7]. However, the implementation of a universal quantum computer capable of performing useful calculations is challenging since it requires many error-corrected logical qubits that involves overhead in number of physical qubits. To obtain a single logical qubit of reasonable error rate, on the order of 10^3 to 10^4 physical qubits of present coherence rates are necessary. For example, factorizing a 15 bit number using Shor's algorithm, a quantum computer would require up to $\approx 10^7$ physical qubits, dependent on the tolerated error rate and the time of the computation [8].

Near term applications of quantum computation is the simulation of quantum chemistry [9], optimization problems by quantum annealing [10], uncertainty and constrained optimization to financial problems [11]. The most anticipated application is however the simulation of quantum chemistry [12]. As an example, protein complexes such as ferredoxin Fe_2S_2 or the Fenna-Matthews-Olson (FMO) complex are known to mediate energy transfer in many metabolic reactions but are intractable on a classical computer. Based on the FMO complex, the efficiency of light harvesting in photosynthesis has been found to notable exceed the expectation based on classical models, such that a quantum description is likely to be required in order to understand the mechanism [13]. Few examples of analogue quantum simulation are the study of fermionic transport [14], magnetism [15] and a quantum phase transition in the Bose-Hubbard model with cold atoms [16]. Using an array of semiconductor quantum dots a simulation of the Fermi-Hubbard model was performed and the simulation of a quantum magnet [17] and Dirac equation was demonstrated using trapped ions [18]. Digital simulation schemes with superconducting devices were demonstrated for spin systems [19].

From the experimental point of view, there are two approaches for quantum simulations: an analog quantum simulator and a digital quantum simulator. The principle of designing an analog quantum simulator is to engineer a quantum system having a controllable Hamiltonian H_{sim} , which can replicate a potentially hard-to-study Hamiltonian H_{sys} , provided there exists a mapping between the H_{sys} and H_{sim} and vice versa.

A digital quantum simulator on the other hand is universal, and would have the capacity to solve a wide range of Hamiltonian's. In digital quantum simulation, one can break the Hamiltonian into gates that are applied in a time-dependent manner. In principle, any model that can be mapped onto a spin-type Hamiltonian can be encoded in a digital quantum simulator. Such a system might be experimentally challenging compared to analogue quantum simulators, in the long run it would be advantageous [19].

This thesis focuses on **Superconducting circuits**, one of the promising candidates for building a scalable quantum computer. The field of superconducting circuits has started in the 1980's with the goal of becoming a competitor in the race to build a universal quantum computer. The key element in superconducting quantum circuits is a Josephson junction. A Josephson junction [20] is a non-linear, dissipation-less element that connects two superconducting islands by either insulating barrier or a metallic barrier. The initial experiment that opened the possibility to use the macroscopic quantum states in a Josephson-junction based superconducting circuits was the discovery of the quantum tunneling effect [21].

Superconducting quantum systems feature individual control, readout and frequency tunability and their properties are rather straightforward to tailor by circuit design [22]. During the past two decades, superconducting qubits experienced a rapid improvement of their coherence properties allowing for demonstration of several major milestones in the pursuit of scalable quantum computation [23]. Such as the control and entanglement of multiple qubits [24], quantum supremacy [25], implementation of a quantum error correction scheme [26], the demonstration of quantum algorithms [27] and encoding quantum information in complex cavity states [28].

In this context, I present the numerical simulations to realize a dipolar analog quantum simulator using an array of 3D transmon superconducting qubits [29]. 3D transmon qubits have a naturally occurring dipolar interaction [30]. One can utilize this interactions by realizing an interacting spin system which opens the way toward the realization of a broad class of tunable spin models in both two- and one-dimensional geometries [31].

One way of classifying superconducting quantum circuits is based on the non-linearity of the system and it's quality factor (Q) as shown in the figure 1.1 [32]. For small anharmonicity and small Q's, devices such as amplifiers(JBA [33], JPC [34], DJJPA [35], JPA [36]), circulators [37] etc. have been already in use and rigorous research is inplace to have better performance. The superconducting quantum bits have higher anharmonicity and higher Q's [22]. In the very little explored intermediate regime, the anharmonicity of devices is approximately equal to the quality factor Q's.



Figure 1.1: Different regimes of superconducting circuits as a function of the relative anharmonicity, and the quality factor. Here α_K is the anharmonicity and Δ corresponds to the detuning frequency. Figure taken directly from [32].

The experimental part presented in this thesis focuses particularly in the intermediate regime, engineering a novel device using Josephson junctions array for QNDmeasurement on a qubit (a key essential for building an analogue quantum simulator). Josephson junctions arrays have been widely used in practical applications, such as, the National bureau of standards uses series arrays of up to 1500 Josephson junctions to define the U.S standard volt [38]. JJA's have also been considered in applications as oscillators [39], mixers, JPA's [40] and an ideal candidate for building quantum hybrid systems [41, 42]. Since the characteristic impedance of a JJAR is greater than the resistance quantum ($R_Q = h/(2e)^2 \approx 6.5 \text{ k}\Omega$), it is an ideal candidate to implement a so called super-inductance [43].

The initial device characterized in this thesis consists of 1000 Josephson junctions in series. From the initial characterization measurements mentioned in chapter 7 of the thesis, I have acquired knowledge to engineer and characterized a JJAR device for a QND-readout measurement. The device is engineered in the intermediate regime shown in the figure 1.1, having an anharmonicity approximately equal to the linewidth for a particular resonator mode. In this regime bi-stability appears at about few photon's [44].

Non-linear bistable systems for QND-readout scheme have already been demonstrated in the past [33, 45, 46, 47]. However, these devices have some major drawbacks such as: unwanted qubit state transitions during readout [47, 48] and as the resonator is filled with photons to achieve bi-stable hysteresis, it leads to excessive back-action on the qubit [49].

In our approach the modified design JJAR.2.0, utilizes the lowest two resonant modes of the JJAR for QND-readout on a qubit. The first mode of the device is dispersively coupled to the qubit [50]. Due to the cross-kerr interaction [51, 52, 44], the first mode is coupled to the second mode of the JJAR.2.0 while the second mode of the array is decoupled from the qubit. The second mode of the JJAR.2.0 is engineered to have an anharmonicity equal to the linewidth [44]. The pump tone on mode one is used to enable qubit readout via mode two. The pump is tuned on-resonant with the mode one when the qubit is in the ground state. The readout tone is chosen on mode two at a fixed frequency, when the qubit is in ground state low signal transmission is measured on mode two. If the qubit is in the excited state the pump on mode one is off-resonant and mode two is shifted by a few MHz higher in frequency, hence high signal transmission is measured on mode two.

1.1 Outline of the Thesis

This thesis consists of 8 chapters. The story starts with chapter 2, where I will discuss the basic concepts of 'High-Q' microwave resonators and rectangular waveguides. Followed by the theory of Josephson junction arrays. Most of the work during the first year of my PhD consisted of setting up the Kirchmair lab, designing and building the microwave setup, which is the topic of chapter 3. In chapter 4, a brief overview on the fabrication of JJA's coupled to a qubit has been discussed, which is part of my work during the last year of my PhD life. A significant part of my work was to engineer the qubit-cavity and qubit-qubit interaction, the subject matter of chapter 5, which is a detail review on qubit-qubit interaction. From numerical simulations, a novel platform for quantum many-body simulations is proposed [29]. High - Q resonators are important in reading out the state of a qubit. In chapter 6, a detail experimental characterization of high-Q stripline resonators is discussed [53]. In chapter 7, a Josephson junction array

By performing continuous time measurements the switching events between two stable solutions are measured for different drive and readout strengths. Finally, in chapter 8, I present the design and characterization of a novel setup using Josephson junction array resonators, which will be a possible candidate for high-fidelity qubit readout in rectangular waveguides.

Chapter 2

Theory of Circuit QED and Josephson Junction Array Resonators

I start the discussion with the basics of microwave resonators, waveguides which is followed by the theory of Josephson junction array resonators. This theory is used to characterize the device parameters which are discussed in a later chapters of this thesis. An example of a driven mesoscopic nonlinear resonator is a duffing oscillator [54, 55], which has two stable solutions for a given set of parameters. In the last section of this chapter I explain **Kramer's theory** to understand the dynamics of stochastic switching between two stable solutions in a driven nonlinear mesoscopic Josephson junction array resonator.

2.1 Microwave resonators and waveguides

One of the major challenges in the superconducting quantum circuits community is to protect the qubit from external noise and to perform efficient readout. One approach is placing the qubit in a three dimensional rectangular waveguide or cavity. Thus the superconducting qubit is shielded inside the cavity environment[30]. In this section, I will discuss microwave resonators, and focus mainly on rectangular cavities and the rectangular waveguide used in our experiments discussed in the following chapters of this thesis.

2.1.1 Rectangular cavities

A three-dimensional rectangular cavity fig. 2.1 has an inner volume V = abd. It forms a rectangular microwave resonator, where a, b and d are the dimensions of the walls. The cavity inner wall width a and the width b determines the fundamental mode frequency TE_{101} of the resonator given by the formula (2.1) [56]. Microwave cavities can be modeled by LC- circuits with resonance frequency in the microwave regime [56].

$$f_{mnl} = \frac{C_l}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{l}{d}\right)^2} \tag{2.1}$$



Figure 2.1: Two half's of a rectangular copper cavity, a, b and d are the width, depth and the height of the cavity respectively.

Here C_l is the speed of the light in vacuum, and the integers m, n, l are the anti-nodes of the standing electric field (2.2) inside the cavity along the x, y and z axis respectively.

One of inner cavity dimension is smaller compared to the other dimensions, which leads to the field becoming constant along the third dimension. A fundamental mode has one of the integer (l) set to zero, and the other two integers (m and n) are set to one. In our experiments, the resonator is designed to have the fundamental mode frequency $\omega_r = 2\pi f_r$ in the range of a few GHz and the higher modes are far detuned from the fundamental mode resonance frequency. To achieve a well defined frequency spacing between the modes of the cavity, similar dimensions are used along the two walls of the cavity, (a = 25 mm, b = 25 mm, d = 10 mm). These will help to avoid unwanted interaction between the qubits and higher resonant modes of the cavity.



Figure 2.2: Electric field intensity inside the cavity. a) First mode of the cavity having a maximum electric component at the center TE_{101} (in our case the fundamental frequency of cavity is $\omega_1/2\pi = 8.504$ GHz). b), c) and d) Electric field intensity for the second, third and fourth mode of the cavity ($TE_{201}, \omega_2/2\pi = 13.51$ GHz, $TE_{021}, \omega_3/2\pi = 13.54$ GHz, $TE_{110}, \omega_4/2\pi = 16.1$ GHz).

The electric field inside the rectangular cavity shows a cosine behaviour as shown in figure 2.2a with maximum at the centre of the resonator for the fundamental mode. The field inside the cavity induces the currents in the cavity walls. These currents oscillate at the same frequency as the microwave field, having the field inside cavity alive.

2.1.1.1 Dissipation and participation ratio

The quality factor of a resonator (Q) sets the ultimate limit for the resonators performance as a quantum memory or bus. The quality factor is defined as the ratio of the amount of energy stored in the system and the amount of energy dissipated. It is given by formula (2.2)

$$Q = \omega \frac{Total \ energy \ stored}{Total \ energy \ dissipated} = \omega T_1 \tag{2.2}$$

Here T_1 is the energy decay and is inversely proportional to the energy decay rate $(T_1 = 1/\kappa)$. The source of dissipation in a microwave resonators can be due to many sources, such as dielectric loss, conductor loss and seam loss [57]. One way of quantifying the losses in any cavities is by using participation ratios [58]. These ratios can be useful in understanding the limitations in performance. The participation ratio p_n is defined as

$$p_n = \frac{Amount \ of \ energy \ sensitive \ to \ loss \ mechanism}{total \ energy \ stored}$$
(2.3)

In most of our experiments, the cavities are made of ultra pure aluminium and typically have a quality factor of 10^6 . Cavities made out of oxygen free copper typically have a quality factor of 10^4 . Rectangular cavities are engineered having at least one seam. A solid block of metal is cut into two half's and a half cavity is milled into each of them as shown in figure 2.1. Both milled blocks are then put together with indium to form the actual microwave cavity. However, the seam causes dissipation thus decreasing the cavity's quality factor [57]. A solution to avoid the seam losses is by using $\Lambda/4$ coaxial resonators [59].

2.1.2 Rectangular waveguides

The main objective of the waveguide is to guide electromagnetic energy. Waveguides are transmission lines commonly used at microwave frequencies. In our experiments, a waveguide can be described as a hollow tube with perfect electrical conducting walls usually filled with vacuum ($\epsilon_r = 1$) shown in figure 2.3. A rectangular waveguide supports only TE and TM modes but not TEM modes.



Figure 2.3: Geometry of a rectangular waveguide. The hollow region can be filled typically with a vacuum.

Let us now consider a rectangular waveguide with dimensions 0 < x < a, 0 < y < b and a > b. There are two type of waves that can propagate, transverse electric waves (TE - waves) and transverse magnetic waves (TM - waves). It is assumed that the waveguide walls are perfect electrical conductor (PEC), such that no losses are present and the propagation constant becomes $\gamma = \beta$. Where β is the propagation constant. It is also assumed that the wave propagates along the z- coordinates which is infinitely long and the electric (\vec{E}) and magnetic fields (\vec{H}) are harmonic in time.

$$\vec{E}(x,y,z,t) = \vec{E}(x,y,z,t)e^{i\omega t}$$
(2.4)

and

$$\vec{H}(x,y,z,t) = \vec{H}(x,y,z,t)e^{i\omega t}$$
(2.5)

Since the field along the x, y plane is independent from the z- position, the electric field can be split in transverse, $\vec{e}(x, y)$ and longitudinal, $\vec{e}_z(x, y)$, components [53]. Thus the field is not depend on the z position. The electric and magnetic fields inside the waveguide can be written as

$$\vec{E}(x,y,z) = [\vec{e}(x,y) + \hat{z}e_z(x,y)]e^{-i\beta z}$$
(2.6)

$$\vec{H}(x,y,z) = [\vec{h}(x,y) + \hat{z}h_z(x,y)]e^{-i\beta z}$$
(2.7)

Assuming the waveguide is source free, and using Maxwell's equations we can re-write [56].

$$\nabla X \vec{E} = -i\omega\mu \vec{H} \tag{2.8}$$

$$\nabla X \vec{H} = i\omega\epsilon\vec{E} \tag{2.9}$$

Inserting the $e^{-i\beta z}$ dependence z, the following relations are obtained

$$E_x = \frac{-i}{k_c^2} \left(\beta \frac{\partial E_z}{\partial x} + \omega \mu \frac{\partial H_z}{\partial y} \right)$$
(2.10)

$$H_x = \frac{i}{k_c^2} \left(\omega \epsilon \frac{\partial E_z}{\partial y} - \beta \frac{\partial H_z}{\partial x} \right)$$
(2.11)

$$E_y = \frac{i}{k_c^2} \left(-\beta \frac{\partial E_z}{\partial y} + \omega \mu \frac{\partial H_z}{\partial x} \right)$$
(2.12)

$$H_y = \frac{-i}{k_c^2} \left(\beta \frac{\partial H_z}{\partial y} + \omega \epsilon \frac{\partial E_z}{\partial x} \right), \qquad (2.13)$$

Where $k_c = \sqrt{\left(\frac{m\pi}{2}\right)^2 + \left(\frac{n\pi}{2}\right)^2} = k^2 - \beta^2$ is defined as the cutoff wave number, and $k = \omega \sqrt{\mu \epsilon}$ is the wave number of material filling the waveguide. For transverse electric waves the electric fields along the z- axis is zero $(E_z = 0)$. Yhe magnetic field along the z- axis is zero $(H_z = 0)$ for transverse magnetic waves. Both waves have to satisfy the Maxwell's equations and the boundary conditions. The boundary conditions are the tangential components of the electric field are zero at the boundaries.

2.1.2.1 TE modes

In the case of TE modes, $E_z = 0$ while $H_z \neq 0$. Equation 2.11 and the equivalent expression for the y component have to be solved, to obtain expressions for the fields. [56]

$$\nabla^2 H_z + k^2 H_z = 0$$

$$\frac{\partial H_z}{\partial x}(0, y, z) = \frac{\partial H_z}{\partial x}(a, y, z) \frac{\partial H_z}{\partial y}(x, a, z) = \frac{\partial H_z}{\partial y}(x, b, z) = 0$$
(2.14)

There are infinitely many solutions to these equations

$$H_{zmn}(x, y, z) = h_{mn} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-jk_z z}$$
(2.15)

The m, n values can take the values m = 0, 1, 2... and n = 0, 1, 2..., but $(m, n) \neq (0, 0)$. The spatial dependence of these components are given by

$$E_x \approx \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-jk_z z}$$

$$E_y \approx \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-jk_z z}$$

$$H_x \approx \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-jk_z z}$$

$$H_y \approx \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-jk_z z}$$
(2.16)

Each of these components satisfy the Helmholtz equation and the boundary conditions. The electromagnetic field corresponding to (m, n) is called a TE_{mn} mode. Thus there are infinitely many TE_{mn} modes. k_z is the z- component of the wave vector. For a given frequency the wave vector is given by

$$k_z = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \tag{2.17}$$

This means that for m and n values such that $k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2 > 0$ or, $f > \frac{c_l}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$, k_z is real and the TE_{mn} mode is propagating.

For *m* and *n* values such that $k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2 < 0$ or, $f < \frac{c_l}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$, k_z is imaginary and the TE_{mn} mode is a non-propagating mode. For a TE_{mn} mode the cut-off frequency is the frequency for which $k_z = 0$. Modes above the cut-off frequency of the waveguide propagate, and the modes below the cut-off frequency have an evanescent field. The cut-off frequency for the TE_{mn} rectangular waveguide is given by:

$$f_{cmn} = \frac{c_l}{2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \tag{2.18}$$

The fundamental mode of a waveguide is the mode that has the lowest cut-off frequency. For a rectangular waveguide it is the TE_{10} mode that is the fundamental mode. It has $f_{c10} = \frac{c}{2a}$. The electric field of the fundamental mode is given by $\mathbf{E} = E_0 \sin\left(\frac{\pi x}{a}\right) e^{-jk_z z} e_y$. In the experiments I discuss in later chapters, the rectangular waveguide is designed to have a cut-off frequency of $f_{c10} = 6$ GHz.

2.2 Theory of Josephson junction

In this section, I will start by a general introduction to the physics of Josephson junctions, followed by the theory of Josephson junction arrays. To understand the properties of JJA, it is easy to start the discussion with a single Josephson junction.

In 1962, [60, 20] Josephson predicted the coherent tunneling of cooper pairs through a thin insulating barrier separating two superconductors as shown in schematic 2.4. The amplitude of the supercurrent that is flowing between the two superconductors depends on the phase difference φ of the two superconducting wave functions left and right of the barrier.

$$I = I_C \sin \varphi \tag{2.19}$$

Where I_c is the maximum supercurrent that can flow through the junction, called critical current of the junction. Equation 2.19 is known as the first Josephson equation [20]. For



Figure 2.4: Schematic of a Josephson junction: A Josephson junction is an insulating layer between two superconductors.

SIS (superconductor-insulator-superconductor) tunnel junctions, the critical current is related to the normal state resistance R_N of the junction and to the superconducting gap of the electrodes $\Delta = \Delta(T)$ via the Ambegaokar-Baratoff relation 2.20 [61].

$$I_C R_N = \frac{\pi \Delta}{2e} \tanh\left(\frac{\Delta}{2k_B T}\right) \tag{2.20}$$

Where T is the temperature, e the elementary charge and k_B the Boltzmann constant, Close to T = 0 equation 2.20 simplifies to

$$I_C R_N = \frac{\pi \Delta_0}{2e} \tag{2.21}$$

Where Δ_0 is the superconducting gap at zero temperature.

If the voltage drop over the junction exceeds $\frac{2\Delta}{e}$, Cooper-pairs can be broken into quasiparticles so that a normal current can flow through the junction.

If there is a finite voltage drop at the junction, the superconducting phase evolves in time according to

$$\frac{d\varphi}{dt} = \frac{2e}{\hbar}V\tag{2.22}$$

Equation 2.22 is generally referred to as the second Josephson equation. By equation 2.19, this phase evolution gives rise to an oscillating supercurrent.

$$I(t) = I_C \sin\left(\frac{2eV}{\hbar}t\right) \tag{2.23}$$

To understand the AC response of a Josephson junction to a voltage bias, the time derivative of equation 2.19 and using equation 2.22 to obtain

$$\frac{dI}{dt}\frac{\hbar}{2eI_C\cos\varphi} = V \tag{2.24}$$

Equation 2.24 is equivalent to the current-voltage relation of an inductance L given by

$$L = \frac{\hbar}{2eI_C} \frac{1}{\cos\varphi} = L_J \frac{1}{\cos\varphi}$$
(2.25)

Where the Josephson inductance is given as $L_J = \frac{\hbar}{2eI_C}$. The energy stored in a Josephson junction can be found by integrating the power from t = 0, the time where ramping of the current is started, to $t = \tau$ where the ramping is stopped

$$E = \int_0^\tau I(t)V(t)dt \tag{2.26}$$

Using Eqs. 2.19 and 2.22,

$$E = \int_0^\tau I_C \sin\varphi \frac{\hbar}{2e} \frac{d\varphi}{dt} dt = \frac{\hbar I_C}{2e} \int_0^\phi \sin\varphi d\varphi = E_J (1 - \cos\phi)$$
(2.27)

where $\phi = \varphi(\tau)$ is the final phase difference over the junction. The Josephson energy $E_J = \frac{\hbar}{2e} \frac{1}{L_J}$ is one of the important energy scales that will be shown to determine the dynamics of Josephson junctions.

A second important energy arises from the electrostatic energy of the Cooper-pairs in the leads forming the Josephson junctions. The two electrodes of a SIS-junction form a capacitor that enables the junction to carry a charge $Q_c = CV$ under a voltage bias V. The Josephson capacitance C defines the electrostatic energy

$$E_{ES} = \frac{Q_C^2}{2C} = \frac{Q_C^2}{e^2} E_C \tag{2.28}$$

where $E_C = \frac{e^2}{2C}$ is the charging energy of a Josephson junction. The junction capacitor, together with the Josephson inductance L_J , forms a LC-resonant circuit with a resonance frequency

$$\omega_p = \frac{1}{\sqrt{L_J C}} = \frac{1}{\hbar} \sqrt{8E_J E_C} \tag{2.29}$$

often referred to as the junctions plasma frequency.

2.2.1 Influence of external fluctuations on Josephson junctions

A Josephson junction also suffers from influence of external fluctuations, such as fluctuations induced by its electromagnetic environment [62, 63]. In the circuit shown in figure 2.5, a current biased junction with Josephson energy E_J and junction capacitance C is subject to phase fluctuations $\delta \varphi$ created by the impedance $Z(\omega)$. Adding the fluctuations to the phase in 2.27 and performing a time average, the energy of the Josephson junction yields

$$E = E_J \cos\left(\varphi_o + \delta\varphi(t)\right) = E_J \cos\left(\cos\Delta\varphi(t)\right)/t = E_J^* \cos\varphi_0 \tag{2.30}$$

where the term $-E_{J0} \sin \varphi_0 \langle \sin \Delta \varphi(t) \rangle$ averages to zero. The Josephson energy E_J is effectively reduced by the fluctuation introduced by the impedance $Z(\omega)$. As $E_J \propto I_C$, the critical current I_C will be affected

$$I = I_C \sin(\varphi_0 + \delta\varphi(t)) = I_C \sin\varphi_0 \left\langle \cos\delta\varphi(t) \right\rangle / t = I_C^* \sin\varphi_0 \tag{2.31}$$

and the term $I_c \cos \varphi_0 \langle \sin \delta \varphi(t) \rangle$ also averages to zero. Hence the presence of fluctuations in the environment of a junction effectively re-normalizes the Josephson energy to $E_J^* = E_J \langle \cos \delta \varphi(t) \rangle$ and the critical current to $I_C^* = I_C \langle \cos \delta \varphi(t) \rangle$.



Figure 2.5: A Josephson junction with Josephson energy E_J and junction capacitance C in a current biased circuit with bias current I_b . The circuit also includes an impedance $Z(\omega)$ which creates a current noise δI

2.3 Josephson junction arrays

In this section I will extend the discussion from single junctions to an array of Josephson junctions, assuming all junctions in the chain to be identical [64, 51, 43]. Since the Josephson junction arrays are designed to have a large Josephson energy, charging effects as well as quantum and thermal fluctuations can be neglected. The whole array can be described by a global phase and charge variable (adopted from chapter 3 of [62]).

To derive the energy-phase and current-phase relations [65], I will consider a Josephson junction chain of identical junctions with Josephson energy E_J and a charging energy E_C with an applied phase bias δ as depicted in figure 2.6. Further assuming $E_J \gg E_C$



Figure 2.6: Chain of N Josephson junctions with applied phase bias δ . The junctions are assumed to be identical.

and $E_J \gg k_B T$. In equilibrium the phase δ will drop uniformly across the individual junctions.

$$\varphi_i = \frac{\delta}{N} \tag{2.32}$$

The total energy of the chain is obtained by summing over the Josephson energies of the individual junctions

$$E = \sum_{i} E_J (1 - \cos \varphi_i) = N E_J \left(1 - \cos \frac{\delta}{N} \right)$$
(2.33)

2.4 Dispersion relation in Josephson junction Arrays

In this section I will derive the dispersion relation of extended plasma resonances in Josephson junction array resonator. The circuit considered is shown in figure 2.7 [51, 52, 66, 67], a Josephson junction array resonator is modeled by a series of parallel LC - circuit of Josephson inductance L_J and junction capacitance C_J . The LC- resonators of the individual junctions are connected to each other via superconducting islands with a small ground capacitance C_0 . The whole circuit shown in figure 2.7 and resembles a simple transmission line, when the charging energy of the Josephson junction is set to $C_J = 0$. For the moment let's consider a chain of infinite length, and fluxes on the superconducting islands Φ_x as coordinates. The island flux Φ_x is related with the superconducting phase φ of the island via $\varphi = \frac{2\pi}{\Phi_0^{\pi}} \Phi_x$. where Φ_0^x is the superconducting flux quantum [51, 43].

The Kirchhoff's law for current conservation for each island of the array using these coordinates is given by equation 2.34. To simplify the treatment lets consider the circuit



Figure 2.7: Circuit diagram considered for the derivation of the plasma resonance of a Josephson junction chain. The junctions are modeled by a series of LC- circuits formed by the Josephson inductance L_J and the junction capacitance C_J . The plasma resonances get coupled in the presence of the ground capacitance C_0 of the superconducting islands.



Figure 2.8: Current conservation for a superconducting island. The directions of the currents are indicated by arrows.

shown in figure 2.8.

$$\frac{1}{L_J}(\Phi_{x-1} - \Phi_x) + (\ddot{\Phi}_{x-1} - \ddot{\Phi}_x)C_J - \frac{1}{L_J}(\Phi_x - \Phi_{x+1}) - (\ddot{\Phi}_x - \ddot{\Phi}_{x+1})C_J + C_0\ddot{\Phi}_x = 0 \quad (2.34)$$

We can solve eq. 2.34 by making a general plane wave ansatz for the flux on the islands

$$\Phi_x = A e^{i(\omega t - kx)}.$$
(2.35)

Using 2.34 and 2.35, we obtain

$$\frac{-2}{L_J} + \frac{-2}{L_J}\cos kx - \omega^2 2C_J + \omega^2 2C_J\cos kx + \omega^2 C_0 = 0$$
(2.36)

which yields the dispersion relation

$$\omega(k) = \frac{1}{\sqrt{L_J C_J}} \sqrt{\frac{1 - \cos kx}{1 - \cos kx + \frac{C_0}{2C_J}}}.$$
(2.37)

The dispersion relation 2.37 is plotted in figure 2.10. The dispersion relation shows two distinct regimes. At low k-vectors the dispersion relation grows linearly with k like the dispersion relation of a transmission line. For large k-vectors the dispersion relation saturates at the plasma frequency of the single junctions $\omega_{plasma} = \frac{1}{\sqrt{L_J C_J}}$. Josephson junction chains show a low phase velocity compared to conventional waveguides in the microwave regime such as coaxial cables, microstrip or co-planar waveguides. For more information please refer to [43].

2.4.1 Modification of JJ array frequencies by adding an extra shunt capacitance

In this section I will discuss the influence of the capacitively shunted Josephson junction array resonator as shown in figure 2.9. The capacitance pads at the end of the JJ array lowers the eigenmode frequencies. Circuit 2.9 can be treated as a single transmission line



Figure 2.9: a) Circuit representation of a capacitively loaded Josephson junction array resonator. C_J is the Josephson junction capacitance, L_J is the Josephson inductance, C_0 is the ground capacitance of Josephson junction and C_S is the shunt capacitance. b) Transmission line model for k modes of a N-junction capacitively loaded JJA. In this case the junction capacitance is not taken into account.

resonator (neglecting the Josephson junction capacitance C_J) [43]. The lower eigenmode frequencies of the capacitively shunted Josephson junction array are more influenced by the shunt capacitance. If the transmission line is cut in to half, and the impedance



Figure 2.10: Dispersion relation as a function of the wave vector k for a unloaded (blue dots) and capacitively loaded C_S (red dots) to a Josephson junctions array.

of both the left and right halves is compared, the situation $Im[Z_{left}] = Im[Z_{right}^*]$ corresponds to an odd k mode resonance. Additionally, from symmetry we have $Z_{left} = Z_{right}$ (always true). Since the impedance's are strictly imaginary, the impedance looking into the resonator section must be zero on odd k resonances. The input impedance is given as (taken directly from [43]).

$$Z_{in} = Z_k \frac{(j\omega_k C_s)^{-1} + jZ_k tan(\beta_k N/2)}{Z_k + j(j\omega_k C_S)^{-1} tan(\beta_k N/2)} = 0$$
(2.38)

Here the propagation constant is approximated by $\beta_k = \omega_k / \nu_k^0$. The odd mode frequencies of the modified JJA using the impedance $Z_{in} = 0$ is given as (equation taken directly from [43])

$$(\omega_k C_S Z_k)^{-1} = \tan\left(\frac{\omega_k}{\omega_k^0} \frac{k\pi}{2}\right)$$
(2.39)

where ω_k is the mode frequencies of the modified JJA, $Z_k = \frac{1}{2} \sqrt{\frac{L_J}{C_J}}$. The even mode frequencies of modified JJA using the admittance $Y_{in} = 0$, and is given as (equation taken directly from [43])

$$-\omega_k C_S Z_k = \tan\left(\frac{\omega_k}{\omega_k^0} \frac{k\pi}{2}\right) \tag{2.40}$$

2.5 Kerr effect in Josephson junction arrays

In this section I will first derive the effective Hamiltonian of the Josephson junction arrays in the linear limit [51, 52]. The non-linearity of the Josephson junctions is then reintroduced as a second order perturbation to this linear Hamiltonian in a later section. In this way we derive the Kerr coefficients of the Josephson junction arrays.

2.5.1 Derivation of Kerr coefficients

2.5.1.1 Hamiltonian

To derive the Hamiltonian of the circuit shown in figure 2.9, lets start by writing down the Lagrangian of the circuit. The Lagrangian is given by

$$\mathcal{L} = \frac{C_S}{2} \dot{\Phi}_0^2 + \sum_{x=1}^{N-1} \left(\frac{C_0}{2} \dot{\Phi}_x^2 \right) + \sum_{x=0}^{N-1} \frac{C_J}{2} (\dot{\Phi}_{x+1} - \dot{\Phi}_x)^2 - \sum_{x=0}^{N-1} E_J \left(1 - \cos\left(\frac{2\pi}{\Phi_0^*} (\Phi_{x+1} - \Phi_x)\right) \right)$$
(2.41)

Where Φ_x is the flux on islands. In order to obtain the Hamiltonian the conjugate momenta (charges) to the fluxes on the defined islands Φ_x are derived

$$Q_0 = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}_0} = (\dot{\Phi}_1 - \dot{\Phi}_0)C_J + \dot{\Phi}_0 C_0 + \dot{\Phi}_0 C_S$$
(2.42)

$$Q_N = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}_N} = \dot{\Phi}_N C_0 + (\dot{\Phi}_N - \dot{\Phi}_{N-1}) C_J$$
(2.43)

$$Q_x = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}_x} = \dot{\Phi}_x C_0 + (\dot{\Phi}_x - \dot{\Phi}_{x-1})C_J - (\dot{\Phi}_{x+1} - \dot{\Phi}_x)C_J$$
(2.44)

One can rewrite the charges in a matrix representation

$$\vec{Q} = \hat{C}\vec{\dot{\Phi}} \tag{2.45}$$

With the derivative of the flux vector with respect to time and the capacitance matrix

$$\hat{\Phi}^{T} = (\dot{\Phi}_{0}, \dot{\Phi}_{1}, \dots, \dot{\Phi}_{N})$$

$$\hat{C} = \begin{pmatrix} C_{0} + C_{j} + C_{S} & -C_{J} & 0 & \cdots & \\ & -C_{J} & C_{0} + C_{J} & -C_{J} & 0 & \cdots & \\ & 0 & -C_{J} & C_{0} + C_{J} & -C_{J} & 0 & \cdots & \\ & \vdots & 0 & \ddots & \ddots & \ddots & \ddots & \\ & 0 & 0 & \vdots & -C_{J} & -C_{J} + C_{0} \end{pmatrix}$$

With this and the inverse inductance matrix

$$\hat{L}^{-1} = \begin{pmatrix} \frac{2}{L_J} & \frac{-1}{L_J} & 0 & \cdots & \\ \frac{-1}{L_J} & \frac{2}{L_J} & \frac{-1}{L_J} & 0 & \cdots & \\ 0 & \frac{-1}{L_J} & \frac{2}{L_J} & \frac{-1}{L_J} & 0 & \cdots & \\ \vdots & 0 & \ddots & \ddots & \ddots & \ddots & \end{pmatrix},$$
(2.46)

Where $L_J = (\hbar/2e)^2(1/E_J)$, the Lagrangian equation 2.41 can be rewritten as

$$\mathcal{L} = \frac{1}{2}\vec{\Phi}^T \hat{C}\vec{\Phi} - \frac{1}{2} \left(\frac{\hbar}{2e}\right)^2 \vec{\Phi}^T \hat{L}^{-1}\vec{\Phi}, \qquad (2.47)$$

By performing the Legendre transformation using the momentum vector

$$\vec{Q}^T = (Q_0, Q_1, Q_2, ..., Q_N) \tag{2.48}$$

We obtain the Hamiltonian of the Josephson junction chain in the linear limit

$$H = \vec{Q}^T \vec{\Phi} - \mathcal{L} = \vec{Q}^T \hat{C}^{-1} \vec{Q} - \frac{1}{2} \vec{Q}^T \hat{C}^{-1} \vec{Q} + \frac{1}{2} \left(\frac{\hbar}{2e}\right)^2 \vec{\Phi}^T \hat{L}^{-1} \vec{\Phi} = \frac{1}{2} \vec{Q}^T \hat{C}^{-1} \vec{Q} + \frac{1}{2} \left(\frac{\hbar}{2e}\right)^2 \vec{\Phi}^T \hat{L}^{-1} \vec{\Phi}$$
(2.49)

Since the Hamiltonian is quadratic, it can be diagonalized and represented in the form

$$H = \frac{1}{2} \sum_{K=0}^{N-1} \hbar \omega_K a_K^{\dagger} a_K.$$
 (2.50)

Where a_K^{\dagger} and a_K are the creation and annihilation operators of the electromagnetic modes in the Josephson junction array. The frequencies ω_K as function of k constitute the dispersion relation of these modes along the chain 2.37. These eigen modes can be found by solving for the eigenvalues[51]

$$\hat{C}^{-1/2}\hat{L}^{-1}\hat{C}^{-1/2}\vec{\psi}_K = \omega_k^2\vec{\psi}_k.$$
(2.51)

2.5.2 Introducing the non-linearity of Josephson junction by perturbation

In this section, the non-linearity of the Josephson junction is re-introduced as a perturbation to the linear Hamiltonian. Therefore adding the quartic term of the expansion of the Josephson energy [51, 52], as the quadratic part was already taken into account.

$$E_J(1 - \cos\left(\frac{2\pi}{\Phi_0}\Delta\Phi\right)) = 1 - 1 + \frac{1}{2}\frac{4\pi^2}{\Phi_0^2}E_J\Delta\Phi^2 - \frac{1}{24}\frac{16\pi^4}{\Phi_0^4}E_J\Delta\Phi^4$$
(2.52)

Where the phase drop across the Josephson junction $\Delta \Phi_x = (\Phi_{x+1} - \Phi_x)$ and the Φ_x are given by as

$$\hat{\Phi}_x = \sum_{m,j} \hat{C}_{x,m}^{-1/2} \psi_{m,j} \sqrt{\frac{\hbar}{2\omega_j}} \left(\hat{a}_j + \hat{a}_j^{\dagger} \right).$$
(2.53)

The Hamiltonian thus transforms into

$$\hat{H}_{NL} = \hat{H} + \hat{U}_{NL} \tag{2.54}$$

with the nonlinear potential energy

$$\hat{U}_{NL} = -\frac{1}{24} \frac{16\pi^4}{\Phi_0^4} E_J \sum_{x=0}^{N-1} \left[\sum_{y,j} \left(\hat{C}_{x+1,y}^{-1/2} - \hat{C}_{x,y}^{-1/2} \right) \psi_{y,j} \sqrt{\frac{\hbar}{2\omega_j}} \left(\hat{a}_j + \hat{a}_j^{\dagger} \right) \right]^4.$$
(2.55)

By utilizing the RWA and neglecting the terms containing more than two creation or annihilation operators, the Hamiltonian of the Josephson junction array resonator is given by

$$\hat{H}_{NL} = \sum_{j} \hbar \omega_{j}' \hat{a}_{j}^{\dagger} \hat{a}_{j} - \sum_{j} \frac{\hbar}{2} K_{jj} (\hat{a}_{j}^{\dagger} \hat{a}_{j})^{2} - \sum_{\substack{j,k\\(j \neq k)}} \frac{\hbar}{2} K_{jk} \hat{a}_{j}^{\dagger} \hat{a}_{j} \hat{a}_{k}^{\dagger} \hat{a}_{k}$$
(2.56)

Where K_{jj} and K_{jk} are the self and cross-kerr coefficients given as (equation taken directly from [51])

$$K_{jj} = \frac{2\hbar\pi^4 E_J \eta_{jjjj}}{\Phi_0^4 C_J^2 \omega_j^2}$$

$$K_{jk} = \frac{4\hbar\pi^4 E_J \eta_{jjkk}}{\Phi_0^4 C_J^2 \omega_j \omega_k}$$
(2.57)

where

$$\eta_{jklm} = \sum_{x} \left[\sum_{y} \left(\left(\sqrt{C_J} \hat{C}_{x,y}^{-1/2} - \sqrt{C_J} \hat{C}_{x-1,y}^{-1/2} \right) \psi_{y,j} \right) \sum_{y} \left(\left(\sqrt{C_J} \hat{C}_{x,y}^{-1/2} - \sqrt{C_J} \hat{C}_{x-1,y}^{-1/2} \right) \psi_{y,k} \right) \right. \\ \left. \sum_{y} \left(\left(\sqrt{C_J} \hat{C}_{x,y}^{-1/2} - \sqrt{C_J} \hat{C}_{x-1,y}^{-1/2} \right) \psi_{y,l} \right) \sum_{y} \left(\left(\sqrt{C_J} \hat{C}_{x,y}^{-1/2} - \sqrt{C_J} \hat{C}_{x-1,y}^{-1/2} \right) \psi_{y,m} \right) \right]$$

Hence the frequency re-normalization shift is given as

$$\omega'_{j} = \omega_{j} - K_{jj}/2 - \sum_{k} K_{jk}/4$$
(2.58)

2.6 Difference of a driven linear and a non-linear oscillator

2.6.1 Driven linear oscillator

A well-known example of a linear oscillator is shown in figure, where mass m is suspended by a spring. A linear oscillator can oscillate with only one frequency, with different amplitude. The response of a linear oscillator system due to a driving force is sum of two parts [68]:

• A steady state part with the frequency of the driving force. The amplitude is completely determined by the strength of the damping force, based on how far the driving frequency is detuned from the natural frequency, and also on how strong the driving force is.

• A transient part which oscillates at the frequency f_d , which is the frequency that the



Figure 2.11: Illustration sketch of a simple linear oscillator, where mass m is suspended to a spring k.

system would oscillate without any external drive.

In the steady-state part of the motion, the frequency of the driving force and its amplitude depends on the damping. Figure 2.12 a) show how the steady state amplitude depends on the frequency for different values of damping. When the damping is increased, the maximum possible amplitude is decreased. The differential equation for the damped, driven oscillator is given as [68]

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = F(t)/m.$$
 (2.59)

Here x is the displacement of the oscillator from equilibrium, ω_0 is the natural angular frequency of the oscillator, γ is a damping coefficient, and F(t) is a driving force.



Figure 2.12: Oscillation amplitude of a linear and non-linear oscillator as a function of the detuning frequency for increasing pump strengths. a) The shape of a resonance doesn't change with the pump strength. b) The resonance curve bends over as the pump strength increases and has two stable solutions for certain parameters. Two different pump strengths $(n_p = 4, 6)$ is shown in figure, red, blue, blow and magenta in the plot corresponds to the stable solutions and the dotted line between them corresponds to the metastable solution.

2.6.2 Driven non-linear oscillator

Nonlinear oscillators in physics, engineering, mathematical and related fields have been the focus of attention for many years and several methods have been used to find approximate solutions to these dynamical systems. In conservative nonlinear oscillators the restoring force is not dependent on time, the total energy is constant [69, 70] and any oscillation is stationary. An important feature of the solutions of conservative oscillators is that they are periodic and range over a continuous interval of initial values [69]. The nonlinear oscillator is described by a differential equation with third- and fifth-power nonlinearity [70]. One of the simplest nonlinear system is a duffing oscillator, which is a x^3 non-linear system. The equation of motion for a duffing oscillator is given by

$$\ddot{x} + \delta \dot{x} + \beta x = F_0 \cos \theta_v - \alpha x^3. \tag{2.60}$$

Where α is the non-linearity in the system, x is the displacement, k is the spring constant, δ is damping coefficient and F_0 is the magnitude of the driving force. Based on the nonlinearity the α can be either positive or negative. For $\beta = 0$ in eq. 2.60, a bifurcation occurs having two stable solutions. It has two stable states corresponding to the steady state oscillations differing in their amplitude and phase as shown in figure 2.12 b). In our experiments shown in chapter 7, the dynamics of the stable steady state solution is studied by using kramer's model which is discussed in the following section.

2.7 Theoretical Model for calculating the switching Rates (Γ)

In this section, I discuss the details regarding the theoretical model based on Kramer's theory of switching [71], used to obtain the fits in figure 7.10b of the chapter 7. The Hamiltonian considered is given in 2.61 and focuses only on one resonant mode of JJAR (indexed by P in what follows) and include in addition a constant shift due to the cross-Kerr interaction $K_{PR}\bar{n}_R$ with the readout mode (indexed by R):

$$H/\hbar = \sum_{i=P,R} (\omega_i a_i^{\dagger} a_i + \frac{K_i}{2} a_i^{\dagger} a_i a_i^{\dagger} a_i) + K_{PR} a_P^{\dagger} a_P a_R^{\dagger} a_R + K_{RP} a_R^{\dagger} a_R a_P^{\dagger} a_P.$$
(2.61)

$$H/\hbar = (\tilde{\Delta}_{PP} + K_{PR}\bar{n}_R)\hat{a}_P^{\dagger}\hat{a}_P + \frac{K_{PP}}{2}\left(\hat{a}_P^{\dagger}\hat{a}_P\right)^2 + i\eta_P(\hat{a}_P^{\dagger} - \hat{a}_P).$$

$$(2.62)$$

Here, the Hamiltonian is written in a rotating frame with respect to the drive with



Figure 2.13: Schematic of the potential landscape for bistable switching. \bar{n}_P is the mean number of photons in the pump mode, $E_{b,(L,R)}$ is the barrier height of the left and right potential, ω_L is the oscillating frequency of left potential well, ω_R is the oscillating frequency of the right potential well, $\bar{n}_{L,R}$ is the mean number of photons in the left and right potential well and ω_0 is the oscillating frequency at top of both potential wells.

strength η_P and with frequency ω_d and the detuning parameter $\hat{\Delta}_P = \omega_P - \omega_d$ with respect to the bare frequency of the pump mode ω_P . Taking the classical limit of the Heisenberg-Langevin equation for mode \hat{a}_P , the following equation is obtained for the complex amplitude $\alpha_P = \langle \hat{a}_P \rangle$ mode (ignoring noise terms)

$$\frac{d\alpha_P}{dt} = \left[-i(\tilde{\Delta}_{PP} + K_{PP}/2 + K_{PR}\bar{n}_R) - iK_{PP}|\alpha_P|^2 - \frac{\kappa_P}{2} \right] \alpha_P + \eta_P.$$
(2.63)

The steady state solution for this equation, defining $\delta = \tilde{\Delta}_P + K_{PP}/2 + K_{PR}\bar{n}_R$ written in terms of the average photon number $\bar{n}_P = |\alpha_P|^2$ is

$$\bar{n}_P = \frac{\eta_P^2}{\left[\delta + K_{PP}\bar{n}\right]^2 + \kappa_P^2/4}.$$
(2.64)

This equation can have either one or three real roots. When it has three real roots, the system shows bi-stability, two roots are stable solutions and one is metastable. For a
given value of δ and K_{PP} , it has three real solutions when $|\delta| > \sqrt{3}\kappa_P/2$ [72, 73], and the drive strength falls in the range $\eta_- \leq \eta_P \leq \eta_+$, where $\eta_{\pm}^2 = n_{c,\pm} \left(\left[\delta + K_{PP} n_{c,\pm} \right]^2 + \kappa_P^2 / 4 \right)$, with the external photon numbers given by

$$n_{c,\pm} = \frac{-2\delta}{3K_{PP}} \left[1 \mp \sqrt{1 - \frac{3}{4} \left(1 + \frac{\kappa_P^2}{4\delta^2} \right)} \right].$$
 (2.65)

One key ingredient to analyze the switching rates in a bistable system within the Kramers framework is a potential landscape in which the two stable solutions occur as local minima. A simple choice for such a potential is obtained by integrating Eq. 2.64 [72] with respect to \bar{n}_P (and dividing by η_P^2 to make it dimensionless) giving

$$U(\bar{n}_{P}) = \frac{K_{PP}^{2}bn^{4}}{4\eta_{P}^{2}} - \frac{2K_{PP}\delta\bar{n}_{P}^{3}}{3\eta_{P}^{2}} + \frac{1}{2\eta_{P}^{2}}\left(\delta^{2} + \frac{\kappa_{P}^{2}}{4}\right)\bar{n}_{P}^{2} - \bar{n}_{P}.$$
(2.66)

Note that this is not a real potential in the Hamiltonian sense, but a fictitious one for the average photon number \bar{n}_P treated as an independent degree of freedom. The critical points of $U(\bar{n}_P)$, namely points where $dU/d\bar{n}_P \equiv 0$ precisely satisfy Eq. 2.64 and the solutions to the equations identify the extrema of the potential landscape. From the quartic form of the potential, as shown in Fig. 2.13, one can see that in the bistable region the potential has a double well shape with two local minima at $\bar{n}_P = \bar{n}_L, \bar{n}_H$ and the local maxima (top of the barrier between the two wells) at $\bar{n}_P = \bar{n}_0$ (with $\bar{n}_L < \bar{n}_0 < \bar{n}_H$). Let us denote the oscillation frequencies at the bottom of the wells (top of the barrier) as ω_L, ω_H (ω_0). The barrier height between \bar{n}_L (\bar{n}_H) and \bar{n}_H (\bar{n}_L) is given by $E_{b,L} = U(\bar{n}_0) - U(\bar{n}_L)$ $(E_{b,H} = U(\bar{n}_0) - U(\bar{n}_H))$. Given these parameters, Kramers formula [71] predicts that the rate of transition out of the wells are of the form $\Gamma_{L \to H} = \Gamma_{0,L \to H} \exp(-\beta_{eff} E_{b,L}) \ (\Gamma_{H \to L} = \Gamma_{0,H \to L} \exp(-\beta_{eff} E_{b,H})) \text{ with the functional}$ form of the pre-factor decided by the relative strengths of the damping rate and the well frequencies $\omega_{L,H}, \omega_0$. In our treatment note that β_{eff} is an effective dimensionless temperature. In addition in the limit of a two state model with localized states at \bar{n}_L and \bar{n}_H , the average state population will be given by

$$P_L = \frac{\exp(-\beta_{eff}U(\bar{n}_L))}{[\exp(-\beta_{eff}U(\bar{n}_L)) + \exp(-\beta_{eff}U(\bar{n}_R))]}$$
(2.67)

and $P_H = 1 - P_L$. The total switching rate, $\Gamma_{H \to L} + \Gamma_{L \to H}$, is maximised when the energy barriers are the smallest and happens for a symmetric configuration with the same barrier height for both directions which is denoted as E_b .

Chapter 3

Experimental techniques

This chapter describes the experimental setup for our 3D circuit QED experiments and measurement techniques used to characterize the Josephson junction array resonator. The design and assembly of the cryogenic microwave setup was a large part of the work performed during the first year of this thesis. In detail, I will present aspects of the cryogenic environment, followed by a description of the input and output lines with their additional component. In the end I will discuss the experimental setup for characterization of the samples and setup for semi-continuous time measurements to evaluate the stochastic switching between two stable solutions in the bi-stable regime.

3.1 Cryogenic microwave setup

In circuit QED experiments, all the experiments are typically engineered to have transition frequencies ω_{01} in the microwave range 4 GHz to 12 GHz. In order to achieve superconductivity of the devices and to initialise the qubits into the ground state and to prevent spontaneous thermal excitation, it is necessary to cool the devices to a temperature well below the corresponding temperature $T_q = h\omega_{01}/\kappa_B \sim 230$ and 430 mK. The devices are placed inside a dilution refrigerator and thermally anchored to its 10 mK base, and shielded from any thermal radiation. Cooling to this low temperature also brings our quantum electrical circuit into the superconducting state, eliminating resistive dissipation. This is one of the fundamental requirements to enable coherence of the qubit, and to avoid unwanted thermal excitation's in the system. The material of the circuit in our case is ultra-pure aluminium with a critical temperature of $T_C \approx 1.2$ K. The other material of choice for our circuits is the oxygen free copper if we want to apply for thin film magnetic-fields. In the same manner, the 3D cavity acquires a high quality factor only when its interior walls are superconducting, because photons would dissipate rapidly in the walls with a finite conductivity. In addition to the low temperature, our experiments require the interaction of the qubits or Josephson junction arrays with a single photon, i.e. a single quantum of energy, which in effect means controlling microwave signals to extremely low powers, on the order of less than 10^{-17} W. This requires careful designing of appropriate microwave wiring and thermalisation, magnetic shielding, cryogenic filtering, low-noise amplification, up- and down- conversion mixing techniques, and fast data acquisition [74].

3.1.1 Cryostat setup

Cooling a device to roughly 1.2 K is relatively simple, by using liquid Helium (LHe) and pumping on it. Firstly, the boiling temperature of liquid Helium is 4 K, and so any device can be cooled to this temperature by simply immersing it in an Liquid He dewar, typically with a probe stick. By additionally pumping on the liquid helium, the vapour pressure is reduced and the liquid is forced to boil, thereby cooling it down further to reach 1.2 K. To reach even lower temperatures, in the mK regime, requires an adiabatic demagnetization Refrigerator or a dilution Refrigerator [75, 76].

The cryostat used in the Kirchmair lab is a oxford Triton 400 Cryofree Dilution Refrigerator (DR) [75, 76] from Oxford Instruments, shown in figure 3.1b. It is a "dry" fridge using a mechanical Pulse-Tube Cooler for reaching roughly 4 K, as opposed to the older "wet" fridge technology which uses a bath of liquid He to achieve the 4 K stage. Both systems then use a closed circuit $\text{He}^3 - \text{He}^4$ mixture dilution unit to attain 10 mK [76]. The Pulse-Tube technology is significantly cheaper since it does not require the expensive liquid helium bath to run, only electricity. This characteristic eliminates the need to refill the liquid helium manually every couple of days and typically gives the base plate more experimental space.



Figure 3.1: Pictures of the lab and the experimental setup: a) Lab space for dilution refrigerator on the left side (first day of my PhD life). b) Close up view of the open fully assembled fridge showing the different temperature stages. c) One Section of soldered stainless steel input lines mounted on to a copper bracket. d) Picture of input lines mounted to a gold coated oxygen free copper brackets anchored on a fridge plate. The gold coating helps in thermalizing the input-lines.

3.1.2 Heat flow and wiring

The experimental devices are fixed onto the 10 mK base plate of the dilution refrigerator (hereafter simply referred to as 'fridge') and are measured by sending microwave signals via stainless steel coaxial microwave cables from the room-temperature electronics down all the different temperature stages of the fridge, passing through a hermetic vacuum feed through at the top of the fridge which is under high vacuum, see Figure 3.2

In circuit QED experiments, one of the key challenges is to be able populate the microwave cavity or waveguide with a coherent state of a single photon on average or less. It is therefore necessary to firstly shield the system from the radiation of the higher temperature stages. This is achieved by thermal anchoring the microwave coaxial cables at each temperature stage, via attenuators and feed through connectors. Secondly, thermal noise picked up by the propagating signal must be minimised. Indeed, classical control signals generated at room temperature are inevitably accompanied by electrical Johnson-Nyquist noise [77] (created by the charge carriers in the conductor being thermally agitated) along the lines to the resonator. At the same time though, the signals must keep a good signal to noise ratio (SNR) throughout the experiment. It is important to choose a large source voltage and a strong attenuation. The entire signal should be attenuated and filtered at different temperature stages by several orders of magnitude, so the noise picked up along the input route is kept low. The weak fields thereafter exiting the experiment through the output line must be strongly amplified to be detected, up to a factor of 10^8 in power. In parallel to these considerations of noise, the entire wiring must not allow for the transport of more heat than the cooling power of the cryostat can handle at each cooling stage, therefore requiring careful selection of cable materials [78]. The heat load of a structure connecting from one stage (Temperature T_1) to another stage (Temperature T_2) is given by:

$$P = \int_{T_1}^{T_2} k(T) \frac{A}{d} dT$$
 (3.1)

Where k(T) is the temperature dependent heat conductivity, A is the area of the structure and d is the length. In our fridge the heat load per stainless steel cable from 4 K plate to the still plate (1 K) is 6.8 μ W. For total of 12 input lines the total heat load on this stage is about 0.082 mW.

3.1.3 Input lines and attenuation

Considering that the choice of material for the cables is forced upon us by the requirements on heat loads as given in eq. 3.1, the task is then to minimize the noise created



Figure 3.2: a) Picture of the 4 K and 55 K stage of input lines inside the cryostat. The red box on the 4 K plate shows the -20dB attenuation. b) Zoom-in picture of the -20 dB Attenuators at the 4 K plate of the cryostat. c) Picture of the K & L low pass filters after -30 dB attenuators at the base plate. d) Zoom- in picture of the LPF and -30 dB attenuators. e) DC blocks mounted on top of cryostat for isolating the fridge and preventing ground loops.

along the input lines. Microwave control signals directed to the experimental device are generated by a microwave source at a power much higher than room temperature. Hence, the signals start off with a good signal-to-noise ratio (SNR). The RMS voltage created by a noisy resistor R is given by Planck's law of black body radiation

$$V_{noise} = \sqrt{\frac{4\hbar\omega BR}{e^{\hbar\omega/k_B T} - 1}} \tag{3.2}$$

where B is the bandwidth of the system, $\omega/2\pi$ is the centre frequency of the bandwidth, and T is the resistance of the noisy resistor. In the Rayleigh-Jeans approximation, for microwave frequencies in the regime 1-10 GHz with temperatures above 2 K, the condition $\hbar \omega \ll k_B T$ holds and reduces V_{noise} to

$$V_{noise} = \sqrt{4k_B T B R} \tag{3.3}$$

This noise power is independent of frequency and depends linearly on the temperature T. The intensity of black body radiation emitted at 300 K in the range up to 40 GHz is roughly 80 times more intense than the radiation emitted at 4 K. A 20 dB attenuator is mounted (reduces input power by a factor of 100) for each microwave input line at the 4 K stage as shown in figure 3.2a. The noise from room temperature is thereby reduced below the noise generated at 4 K. Another set of 30 dB attenuators is mounted at the base plate of the cryostat, as shown in Figure 3.2b reducing noise below 20 mK. The attenuators also simultaneously serve the purpose of achieving the ultra-low power required to have a single photon interacting with the device.

All the input lines are mounted onto gold plated oxygen free copper brackets at each stage of the cryostat for proper thermalisation. For the purpose of planned experiments, two 'sets' of 6 cable lines have been built in total extending down into the cryostat, as seen in Figure 3.1C. Tube A has six cable lines each with -20 dB attenuation at the 4 K plate of the cryostat and -30 dB attenuation at the base plate of the cryostat. All stainless steel microwave cables have soldered SMA connectors and should present minimal reflections (< -20 dB) at each connector along the line as shown in figure 3.3. When microwave pulses propagate along the cable line and reflections at connectors are significant, then the percentages of the pulse's power being reflected several times will lead to constructive and destructive interference and the original pulse will be distorted and followed by reflected smaller pulses. This is an unwanted effect for controlling the state of a system. Achieving low reflections when soldering SMA connectors is a difficult and tedious task, and it depends on several practical techniques. In short: the centre conductor must not be scratched; the centre pin should not have traces of solder on it after being soldered via the small hole onto the centre conductor; the dielectric of the coaxial cable must be prevented from expanding from the heat applied when soldering the connector onto the outer conductor. An expanded dielectric leads to an unwanted impedance mismatch, which in turn leads to strong reflections. After soldering both connectors of each microwave cable's, its reflection coefficients were measured. These



Figure 3.3: VNA measurements: a) shows the reflection measurements $|S_{11}|$, $|S_{22}|$ for one typical soldered cable between the 100 mK stage and the base plate of the cryostat. Achieving low reflections when soldering SMA connectors is most important. b) Direct transmission measurement of two input lines in one of the microwave sets, with a -20 dB attenuator at the 4 K plate.

measurements are performed with a vector network analyser (VNA) from Keysight. The VNA measures the four S-parameters $(S_{11}, S_{21}, S_{12}, S_{22})$ as $S_{ii} = 10 \log(Pout/Pin)[dB]$.

Figure. 3.2 e) shows the installation of DC blocks on the room temperature plate of the cryostat which is necessary to prevent ground loops. A ground loop occurs when there is more than one ground connection path between different instruments. As there are many different instruments and cables connected to the cryostat, it can easily happen that the grounds of two instruments attached to different power lines are both connected to the cryostat, creating a so called ground loop. Since these two grounds can be on slightly different potentials, equalizing currents will flow along unpredictable paths within the cryostat. These stray currents can create unwanted magnetic fields, being a major problem for flux bias lines and noise that can affect the coherence of devices. Thus the microwave lines going into the fridge are isolate with DC blocks shown in figure 3.2 e), so that the fridge is electrically isolated and only grounded via single conducting ground cable. Nevertheless it is still challenging to get rid of ground loops from other instruments used for measurements [76], especially if DC currents are necessary in the experiments.

3.1.3.1 Calibration of input lines

To precisely achieve the desired low power of the control signal the transmission of the entire input line is measured. For the cryogenic setup shown in fig 3.6 a. As the input lines are frequency dependent, I consider the values between 4 - 8 GHz as that is the typical working regime for most of the experimental designs. The (microwave cables) have a strong frequency dependence of their attenuation per meter, which is about -6 dB/m at 6 GHz.

The plot shown in figure 3.3 b) shows a typical transmission measurement on two cable lines of the input lines, with -20 dB attenuator at the 4 K plate. These results indicate that the transmission through the cryostat at RT behaves as expected and proves that the wiring was successfully installed combined with S_{11} measurements. When cold, the transmission of the input lines within the cryostat might change a bit, as the components and cables will have lower resistance and hence lower attenuation.

3.1.4 Sample thermalisation

Thermalising the samples is a crucial and important step for all the experiments. Devices which are not thermalized can show strong dephasing, suffer from the creation of quasiparticles and thus to dissipation and have reduced energy relaxation times. Hence thorough thermalization of cables, attenuators and microwave components at the various temperature stages of the fridge is not only important for reducing the heat load, but also protecting the devices from thermal radiation.

In our cryostat design, all the input and output lines are thermalised using gold plated copper brackets as shown in figure 3.1 c) and d). The samples are mounted on oxygen free copper T-beam holders which are connected to a copper circular bracket 3.4 c). The copper bracket is bolted against the base plate of the cryostat for a good better thermalisation of samples 3.4 a). Feed through's are mounted on top of the T-beam holder to connect the devices to the input and output lines of the cryostat. Each T-beam holder is designed to hold up to maximum of three experiments in each cool down. A total of 6 feed through's are mounted on top of the figure 3.4 a). The devices are connected to the feed through's with copper microwave coaxial lines soldered with SMA connectors. Optimizing the thermal anchoring requires specific considerations



Figure 3.4: Mounting brackets and shields: a) At the base plate of the cryostat two circular brackets are attached at the bottom of the base plate with vacuum feed-throughs. b) Picture of 3D-rectangular waveguide on a T-beam holder thermally anchored to the base plate of the cryostat. The μ - metal shields is bolted to the circular bracket shielding the samples. c) Schematic Picture of the circular brackets with cooper sample holder shielded with 2 layers of μ - metal shields and a copper or Nb shield as the outer shield.

on heat conduction across solid/solid interfaces. The heat flow across a pressed contact is insensitive to changes in contact area for a given total force pressing the two surfaces together. Indeed, the thermal conductance increases approximately linearly with pressure, as has been observed experimentally [79]. The opposite occurs for solder and glued joints. Hence, a good thermal connection between two solid interfaces requires a strong force, and not a large area. In conclusion, one should minimize the number of thermal interfaces and to maximize the force of bolting. Using brass screws to mount samples on the T-beam will thermalise better than using the stainless steel screws. Since the thermal conductivity of the brass is 6.6 times higher than the thermal conductivity of the stainless steel. Other techniques include using molybdenum washers with stainless screws, as the brass screws are two fragile and can break the threads easily by tightening. Molybdenum washers have higher thermal conductivity (139W/(mK)), effectively compresses the bracket.

3.1.4.1 Magnetic shielding

Shielding the samples is another important aspect in performing circuit QED experiments. As superconducting devices are sensitive to stray magnetic fields, this fields can lead to dephasing and microwave losses. This can be understood in terms of the effect of stray magnetic fields on the quality factor (Q) which is given by [80].

$$Q = \frac{G}{R_s} \tag{3.4}$$

Where, G is the geometric factor of superconducting device and R_s is the surface resistance. R_s can be seen as the contributions from the surface magnetic field and other components. The high stray magnetic field increases the surface resistance, thereby degrading the quality factor (Q). These can be effectively addressed by appropriate use of magnetic shields that reduce the magnetic field in a prescribed region. Figure 3.4 C shows the schematic of two layered amumetal shields, The material we are using is amumetal 4K with a thickness of 1.01 mm. Amumetal 4 K is a high nickel content alloy, having higher permeability at lower temperatures. A single layer of amumetal has a magnetic field attenuation (A) of about 1860 at DC and 350 at 50 Hz. In our case we have two layered amumetal shields which have attenuation of 1200,000 at DC and 43000 at 50 Hz. One can calculate the shielding performance using the following formula 3.5.

$$A = 1 + S_1 + S_2 + S_1 S_2 N_{12}$$

$$S_1 = \mu/4(1 - Ri_1^2/Ro_1^2)$$

$$S_2 = \mu/4(1 - Ri_2^2/Ro_2^2)$$

$$N_{12} = 1 - Ro_1^2/Ri_2^2$$
(3.5)

Here Ri, Ro is the radius of the inner and outer layer of amumetal. On the outside of the amumetal shields oxygen free copper shields or Nb shields are used to thermalise and shield the whole sample holder as shown in figure 3.4 C.

3.1.5 Cryogenic amplification chain

In order to measure the transmitted signal with room-temperature microwave electronics, the average signal ($\bar{n} = 1$ photon) coming out of the devices must be amplified by at least 10⁸ times to attain voltages in the mV regime to be detected by analog-to-digital converter [81]. Thermal photons and amplifier noise are larger than the signal itself, and so along the output line any additional losses of the signal power between the sample and the first amplification at 4 K plate would require significantly more averaging to detect



Figure 3.5: a) A picture of the ultra low noise HEMT amplifier with a gain of 40 dB is mounted on the 4k stage on the output lines. b) Picture of the Quinstar isolators and K & L low pass filters at the base plate on the output stage.

the signal. For example, a loss of 3 dB, in power requires two times more averaging to achieve a good signal-to-noise ratio (SNR).

To avoid the problem of having any losses along the output chain a special type of superconducting cable made out of NbTi is mounted up to first amplifier placed at the 4 K stage. Ultra low noise HEMT (High Electron Mobility Transistor) amplifiers from Low noise factory, designed to operate at cryogenic temperatures have been used. See a picture of it in fig 3.5 a). The 4 - 8 GHz HEMT amplifies the signal by a gain G = 46 dB, and the 1-12 GHz HEMT amplifies the signal by a gain of G = 39 dB and noise temperature of 6 K. The reason behind thermally anchoring the HEMT at 4K plate and not at the base plate is that it dissipates 4 mW of power. The noise output of the HEMT using the noise power generated by a lossy component with equivalent noise temperature T^e and a bandwidth B is given by:

$$N = k_B B T^e \tag{3.6}$$

A electrical component with gain G then amplifies the input noise power N_{in} and also amplifies its intrinsic noise power N_a coming from the noise generated by the component itself. The output noise power N_{out} is then given by

$$N_{out} = GN_{in} + GN_a = Gk_B(B_{in}T^e_{in} + B_aT^e_a)$$

$$(3.7)$$

For example one of our HEMT, has a bandwidth B_a of 4-8 GHz, the noise temperature is specified to be between $T_a^e = 5.5$ k. The input noise source of the amplifier is the noise generated by the coaxial superconducting cable with specified gain of $G_{SC} = +26$ dB over bandwidth of up to $B_{in} = 20$ GHz thermalized at $T_{in}^e = 4$ K. By substituting this values into equation 3.7 with a gain of G = +46 dB, the total output noise at the output of the HEMT amplifier is $N_{OUT}^{HEMT} = 4$ dBm. Further the noise travels up the fridge through the stainless steel coaxial cables and propagates through room temperature cables into the room temperature amplification which has a gain of around +40 dB, with typical noise < 1.2 dB. It is crucial to calculate the noise amplification through this chain in order to make sure the various amplifiers do not get saturated by the noise, i.e. that the amplified input noise for each amplifier does not reach its 1 dB compression point. The total signal-to-noise ratio(SNR) is principally governed by the noise added by the first amplifier when its gain G_1 is large according to Frii's law [56].

$$T_a = T_{a,1} + \frac{T_{a,2}}{G_1} + \frac{T_{a,3}}{G_1G_2} + \dots,$$

With G_i representing the gain of each individual amplifier and $T_{a,i}$ the noise temperature of the *i*'s amplifier. Where the noise temperature $T_{a,i}$ is obtained from its specified noise figure $F_i = 1 + T_{a,i}/T_0$, where T_0 is room temperature and the noise figure f_i is commonly expressed in dB as $NF_i = 10log(1 + T_{a,i}/T_0)$.

3.1.5.1 Isolators

The signal coming out of the cavities or the waveguide is sent through (K&L) 1-12GHz low pass filter connected to a set of isolators as shown in fig 3.5 b). The set of isolators are used to reflect thermal noise coming from the HEMT amplifier. These circulators are passive non-reciprocal three port devices, But the third port of the isolators are terminated by placing 50 Ω and use port 1 as input and port 2 as output. So the noise traveling back down the line from the amplifier enters the isolator at port 2 and is redirected into the absorbing load at port 3, providing an isolation of > 25 dB. On the other hand, the signal coming from the cavity or waveguide traveling up the cryostat to be amplified and detected passes through the isolators with a minimal attenuation of < 0.5 dB. Two such isolators are placed in series to maximise the effect.

3.2 Experimental setups

Figure 3.6 a) shows the experimental setup for measuring the devices mentioned in this thesis. Since the samples are coupled capacitively to the rectangular cavities or waveguides. The scattering matrix S_{11} , S_{12} , S_{21} , S_{22} can be obtained from the direct transmission measurements using a vector network analyzer (VNA). Unlike the impedance and admittance matrices which relates the total voltages and currents at the ports, the scattering matrix relates the voltage waves incident on the ports to those reflected/-transmitted from the ports [56].

A two tone spectroscopy can be seen as a pump probe experiment [43]. A two-tone spectroscopy consists in measuring the resonator transmission using two microwave signals. The first one, the probe tone, comes from the VNA and has a constant frequency. This frequency is the one of the resonator modes of the device. The second one, the pump tone, comes from a microwave source. It varies in frequency with a constant pump strength. When the probe tone frequency matches the pump tone frequency, the probe tone shifts in frequency as the probe and pump resonant modes are coupled to each other. From the transmission and two-tone measurements, the photon number on the resonant modes are calibrated.

3.2.1 Time-domain measurements

This sections deals with the measurement scheme used to measure the stochastic switching between two stable solutions around the bi-stable region in a mesoscopic Josephson junction array resonator. The experimental setup for time-domain measurement is shown in figure 3.6 a), where the VNA ouput is replaced with a signal generator and the VNA input is replaced by an ADC setup. The ADC setup is shown in the schematic 3.6 c). Two microwave sources are used at different frequencies, one for the readout resonant mode and the other source for exciting the resonant mode. Phase stability between the different devices is very important and in our setup the VNA, signal generators and other devices are referred to a rubidium clock distributing 10 MHz as external reference.

For the experiments discussed in this thesis, a down mixer is used instead of an IQmixer. The advantage of the down mixer is the higher bandwidth than the IQ-mixers,



Figure 3.6: Experimental setup. a) Block diagram of our experimental setup for measurements using a Vector network analyzer and two-tone spectroscopy. The total attenuation used along the input lines of the cryostat is -50 dB. The signal coming out of device is amplified and the frequency response of device is measured at the input of a VNA. b) Schematic of an IQ-mixer, I and Q corresponds to the in-phase and quadrature and R is the output signal of the IQ-mixer. c) Time-domain measurement setup: The amplified signal coming out of the cryostat is downmixed to an intermediate frequency ω_{IF} and amplified further with three stages of a standford amplifier (SR830) before recording the signal with ADC acquisition board (SDR 14 from SP devices).

and cheaper compared to the IQ-mixers. A down mixer has three signal ports, these three ports are the radio frequency (RO) input, the local oscillator (LO) input, and the intermediate frequency (IF) output. A mixer takes an RF input signal at a frequency (ω_{RO}) , mixes it with a local oscillator (LO) signal at a frequency (ω_{LO}) , and produce an intermediate frequency (IF) output signal that consists of the sum and the difference frequencies. When the sum frequency is used as the IF, the mixer is called upconverter, when the difference is used the mixer is called a downconverter. The frequencies of the microwave RF source ω_{RO} and the local oscillator ω_{LO} is different and the output of the mixer is given as follows in the equation 3.8.

$$I = C_I' R \cos\left(\Delta \omega t + \Delta \psi\right) \tag{3.8}$$

Here C'_I is the conversion factor, R is the RF input signal and $\Delta \psi$ is the phase difference between the two signals that depend on the length of the cables. *I* is a signal oscillating with the frequency difference $\Delta \omega = \omega_{RO} - \omega_{LO}$ between the microwave RF source and the local oscillator signals. Where as in a IQ-mixer (schematic shown in figure 3.6 b)), the local oscillator signal is split up in a 90 degree hybrid, which are then in phase quadrature. Each of the mixers multiplies then one of the split signal with one of the IF (I and Q) inputs which are then added in the Hybrid without any additional phase shift. So, the output signal at the RF-port can be calculated by assuming three input signals.

$$LO = A_{LO} \cos (2\pi f_{LO}t)$$
$$I = A_I(t) \cos (2\pi f_I t)$$
$$Q = A_Q(t) \cos (2\pi f_Q(t - \phi))$$

The LO-signal with time independent amplitude A_{LO} is first split up into two 90 degree phase shifted signals. The output of each mixer is given by the product of the LO and IF signals. If the two IF signals are in phase quadrature, i.e $\phi = \pi/2$ with the same frequency $F_I = f_Q = f_{IF}$ and assuming the same time dependent amplitude, i.e $A_I(t) = A_Q(t)$ the upper side band at frequency $f_{LO} + f_{IF}$ vanishes. In the second hybrid, the signals of two mixers are combined without any phase shift

$$R(t) = \frac{A_{LO}A_I(t)}{\sqrt{2}}\cos\left(2\pi(f_{LO} - f_{IF})t\right).$$
(3.9)

To understand the stochastic switching between two stable solutions of JJAR, a downmixer and one channel of the acquisition card is used to record the amplitude response of the resonator (since we are mostly interested in the amplitude response).

3.2.2 Data acquisition

In this section I will discuss how the transmitted voltage at the output of the mixer is digitized and how the real time data processing on the measurement computer is done. For the semi continuous measurements, I used the SP devices SDR14 (800 Msamples/s) acquisition board to semi-continuously monitor the amplitude of the transmitted signal. The setup is shown in figure 3.6 c). Two signal generators, one for the readout mode (ω_{R0}) and the other for the excitation mode (ω_P) of the Josephson junction array resonator are used to apply a signal to the input of the device. The RF signal coming out of the cryostat (RF) and the local oscillator reference signal (LO) are down-converted to an intermediate frequency (ω_{IF}) . This is done by mixing the readout signal oscillating at ω_{RO} with a local oscillator (LO) running at $\omega_{LO} + \omega_{IF}$. The signal and the reference are down-converted to an intermediate frequency $\omega_{IF} = 2\pi * 10$ MHz that can be resolved with an ADC. The mixer multiplies the inputs, producing an output signal with components oscillating at the sum $\omega_{RO} + \omega_{IF} + \omega_{RO}$ and difference $\omega_{RO} + \omega_{IF} - \omega_{RO}$ frequencies. The down converted RF signal (RO) and reference (LO) can be written as

$$RO(t) = A_{RO}(t)\cos\left(\omega_{IF}t + \theta_{RO}(t)\right). \tag{3.10}$$

and

$$LO(t) = A_{LO}\cos\left(\omega_{LF}t + \theta_{LO}\right). \tag{3.11}$$

both oscillating at the intermediate frequency.

Further, the signal coming out of the down mixer (ZMX-10G+) is fed through a minicircuits 10 MHz low pass filter that damps out the fast oscillating component leaving behind the intermediate frequency. The signal is later amplified with three stages of a standford amplifier (SR830) with gain of +25 dB and later recorded with ADC acquisition board (SDR 14).

Chapter 4

Fabrication of a Josephson junction array resonator and single junction based transmon qubit

In this chapter I will present the procedure for fabrication of a Josephson junction array resonator and superconducting qubits. The development, implementation and optimisation of micro- and nano-fabrication processes were a central effort for the last chapter of my thesis. It is a crucial part of circuit QED to precisely engineer defined properties of the devices to enable quantum information processing, and this chapter illustrates how to achieve them.

4.1 Fabrication of a JJA coupled to a qubit

The basic general lithographic procedure is presented in Figure 4.1: first, a substrate surface is coated with a radiation-sensitive polymer film (resist) and exposed to radiation in some desired pattern; following exposure, a development step removes the exposed resist, thereby leaving the pattern in relief on the substrate surface; the substrate itself can then be patterned by depositing a metal into the open areas of the resist relief pattern; finally, the resist is removed in the lift-off step and the result is the desired

device. The substrate used for our devices is a high resistive > 10000 Ωcm double side polished intrinsic Silicon substrate from **TOPSIL** with a thickness of 530 μm .

4.1.1 Cleaving and cleaning the substrate

Fabrication of Josephson junctions requires a very clean and precise procedure in order to achieve the long coherence times for the qubits, and high yield for JJA's. A small residue or dirt on the substrate will limit the coherence. As a first step in fab process the 4' substrate is cleaved into 4 quarter's. One quarter of the substrate is later sonicated in Acetone for about 1 - 2 hours followed by isoproponal and distilled water.

4.1.2 Resist

The spin coating is done by depositing droplets of resist onto the chip with a precise pipette, making sure that the resist does not flow over the edge of the chip. Any resist reaching the underside of the chip will prevent the chip from laying flat in the e-beam lithography system later. The first layer of resist spun onto the chip with the spin coater is a pure Copolymer of methyl methacrylate (MMA) and methacrylic acid (MAA), which is 3-4 times more sensitive than a pure poly methyl methacrylate (PMMA). It is very sensitive to the direct, secondary and back-scattered electrons from the beam during lithography. It basically serves as a spacer between the substrate and the second resist layer. The spin coating is done at 500 rpm for 40 s and followed by a dynamic ramp to 1500 rpm for 60 s with a very quick ramp up time. This spinning speed creates a layer of thickness of roughly 250 nm. The wafer is later baked at 200° C for 5 minutes on the hotplate. The second layer is a pure low sensitivity, but high resolution polymethyl methacrylate PMMA950k (a large molecular weight) that is spun directly thereafter at 2000 rpm for 60 s creating a thickness of approx. 550 nm, followed by baking again at 200° for 5 minutes. Figure 4.2 a) shows the thickness of PMMA resist measured using the Ambios profilometer at the center of the substrate. b) shows the total thickness of resists (copolymer and PMAA).



Figure 4.1: Fabrication of the device. a) A layer of co-polymer of thickness 450 nm is spun on the clean silicon substrate and then baked. Followed by the second layer of PMMA of thickness 550 nm which is spun on top of the co-polymer layer and baked again. b) The desired junction pattern is exposed with a 30 KV electron-beam lithography machine. c) The exposed resist is developed away with an IPA: Water mixture in a 1:3 ratio. d) The developed substrate is evaporated with thin film aluminium of thickness 25 nm at a growth rate of 1 nm/s at a vertical angle of 25°. Followed by a static oxidation for 1 min with a pressure of 10 mbar. e) Second angle evaporation of aluminium of thickness 30 nm at an angle of -25° . f) Resist lift-off in acetone for 2 hours at 60° C.



Figure 4.2: Profilometer measurements: a) Thickness of PMMA resist layer, b) bi-layer (copolymer and PMMA) on a silicon substrate. A gentle scratch is made on the substrate to measure the thickness of the PMMA and bi-resist layer. The tip of the profilometer is scanned across the substrate through the scratch, thus measuring the thickness of resist layers.

4.1.3 Lithography

Fabricating Josephson junctions requires electron-beam lithography that allows patterning of features down to roughly a few 10's nm. The e-beam lithography system focuses a beam of electrons directly onto the polymer resist on the chip, resulting in breaking of chemical bonds of the exposed polymer. Introducing a different solubility of the exposed vs unexposed regions of the resist in certain solvents, therefore allowing the removal of the desired exposed pattern in the chosen solvent to form a shadow mask. Therefore, the critical feature of an e-beam system is its acceleration voltage. A low acceleration voltage of e-beam can result in excessively strong back scattering. During the development of Josephson junctions in this chapter, I used a Raith system with an acceleration voltage of 30 kV. For proper focusing using the large and small write field's, I introduce a small scratch on the substrate. Small features in the pattern are exposed using a small aperture (100 μ) with a beam current of 36 pA. Using the small aperture results in high resolution, but slower in exposure time. Whereas large contact pads are exposed with bigger aperture (2 mm) with a beam current of 5 nA. The technique adopted for Josephson junctions arrays is a bridge-free technique [82], where the junction size is of a few μm and for qubit, a cross-junction type junction size of 100 X 100 nm. The bridge is achieved using a bi-layer of resist and harnessing the undercut created by the backscattering of the electrons in the lower resist (due to its higher exposure sensitivity).



Figure 4.3: a) Picture of the Raith 30 kV e-beam lithography system. b) Picture of MEB550S evaporator used for evaporating the aluminium. c) Optical image of the JJAR.2.0 device after e-beam lithography (substrate with bi-layer resists). d) Optical dark field image of the Josephson junction array after e-beam lithography exposure (substrate with bi-layer resists).

Depositing the aluminium film at two different angles with an oxidation step in between creates the junction.

4.1.4 Development

After the electron beam lithography, the exposed pattern is developed, i.e. by carefully dipping and slowly moving the chip in a mixture of IPA: Water (1:3) for 105 s followed by a rinse in distilled water for a few seconds and then a gentle N_2 blow dry. It is important to blow dry at soft pressure in order to avoid breaking of the bridges. The

temperature of the development solvent should be maintained at $6^{\circ}C$. Huge change in the solvent temperature can lead to under or over developed structures.

4.1.5 Double angle evaporation

Double angle evaporation is the common technique to produce Josephson junctions [83, 82, 67]. The technique involves two layers of aluminium in two different steps evaporated under two different angles, with intermediate static oxidation step. The evaporation angle used for fabricating the device is typically 25° . For evaporation of the sample an e-beam evaporator **MEB550S** plassys is used. The main properties of an evaporator is a very stable evaporation rate between 0.2 and 1 nm/ s, a stage that can rotate the sample at any angle between 0 and 180° in both directions, and an inlet for oxygen to perform the oxidation step. The load lock is separated from the main chamber with a gate valve.

The sample holder with a sample is loaded in the load lock chamber. Once the load lock reaches a pressure of $< 10^{-7}$ mbar the sample is rotated at an angle 90° and cleaned for 2 min with Ar flow rate of 10 sccm and oxygen flow rate of 5 sccm. The sample holder is later rotated to an angle of 180° , and Titanium gettering for 2 min to improve the vacuum pressure [84] in the chamber. Then a first layer of 5N pure aluminium is evaporated onto the rotated sample holder under a defined angle of 25° . The deposited aluminium layer forms the bottom electrode of the Josephson junction with thickness of a 25 nm. In the subsequent oxidation step, the sample chamber is filled with oxygen with pressure of 10 mbar for 1 min. The result is a thin aluminium-oxide layer with a thickness on the order of a nanometer depending on the partial oxygen pressure and the oxidation time. The oxidation parameters are controlled by using the calibration results for different test Josephson junctions. After the oxidation, the sample chamber is pump down to a pressure of $< 10^{-7}$ mbar and the sample holder is rotated to a second position 155° relative to the substrate normal. Then the second layer of 30 nm aluminium is evaporated. In this way, the upper aluminium layer which overlaps with the lower aluminium layer forms the top electrode of the Josephson junction.

4.1.6 Lift-off

The final step after the evaporation is to strip the resist bi-layer (and the thin film of aluminium now on top of it) from the silicon substrate in a process called "lift-off". This is done by placing the chip in hot Acetone at 60° for 2-3 hours. At the end of the lift-off, the sample is gently sonicated for 30 sec to get rid of the extra aluminium thin film which sticks on the sample. After dipping in IPA and distilled water the sample is blow-dry with N_2 , and the chip is later inspected under an optical microscope.

Chapter 5

Publication 1: Numerical simulations to realize Dipolar Spin Models with Arrays of Superconducting Qubits

My contribution

I took the leading role in performing the numerical simulations described in this paper and helped in writing the manuscript. From the numerical simulations and with theory group collaborators in Peter Zoller's group, a novel approach for quantum simulation using array of superconducting qubit is proposed and discussed in this paper. This chapter covers the FEM based numerical simulations to realize spin models with arrays of superconducting qubits. The basic building blocks are 3D Transmon qubits, utilizing the naturally occurring dipolar interactions to realize interacting spin systems. This opens the way toward the realization of a broad class of tunable spin models in both two- and one-dimensional geometries. In collaboration with Peter Zoller's theory group we illustrate the potential offered by these systems in the context of dimerized Majumdar-Ghosh-type phases, archetypical examples of quantum magnetism, showing how such phases are robust against disorder and decoherence, and could be observed within state-of-the-art experiments.

5.1 Motivation

In the present work we propose and analyze a novel setup for an analog quantum simulator of quantum magnetism using superconducting qubits. The scheme builds on the remarkable recent developments in Circuit QED [85, 26, 86, 87, 88, 89] in the context of quantum simulation [90, 91, 92, 93, 94], and especially the 3D Transmon qubit [30, 95]. The scheme (illustrated in Fig. 5.1) promises a faithful implementation of many-body spin-1/2 Hamiltonians involving tens of qubits using state-of-the-art experimental techniques. The central idea behind the present work is to exploit the naturally occurring dipolar interactions between qubits to engineer the desired spin-spin interactions. In combination with the flexibility offered by solid-state setups for realizing arbitrary geometry arrangements, this allows us to design general dipolar spin models in ladder and 2D geometries. As we will show, our scheme competes favorably with present and envisaged quantum simulation setups for magnetism with cold atoms and trapped ions [96, 97, 98], and enables us to address some of the key challenges of quantum simulation including equilibrium and non-equilibrium (quench) dynamics [99]. Moreover, we note that exploiting dipolar interactions to design dipolar spin models is conceptually different, and complementary to the remarkable recent experiments with superconducting circuits toward realizing the superfluid-Mott insulator transition, based on wiring up increasingly complex circuits of superconducting stripline cavities [90]

In our analysis we address two of the key aspects of the design of our proposed simulator for quantum magnetism. First, we present a feasibility study of state-of-the-art experimental setups: this includes a discussion of the general mechanism to generate dipolar interactions between 3D Transmons, combined with *ab initio* simulations of the coupling strength in our spin model for various geometries. Second, we illustrate how state-of-the-art setups, composed of up to a dozen qubits and characterized by typical disorder and decoherence rates, are already able to demonstrate paradigmatic signatures of quantum magnetism. In particular, we show how a dimerized phase [100], a valence-bond-solid reminiscent of the Majumdar-Ghosh state widely discussed in the context of quantum spin chains [31], can be realized (via adiabatic state preparation) and probed with current technologies.

The chapter is organized as follows. In Sec. 5.2, we introduce the XY model Hamiltonian, and provide a short summary of the parameter regimes we are interested in. In Sec. 5.3, we describe the circuit QED setup, and present detailed finite-element simulations to access the relevant couplings in the systems, and describe their tunability. In Sec. 5.4, we discuss a simplified circuit model, which provides a generic tool to understand the results in Sec. 5.3. In Sec. ??, we perform a numerical study of the ground state properties of the XY model on a triangular ladder, and discuss the observability of the dimer phase using adiabatic state preparation. Finally, in Sec. 5.6, we draw our conclusions and present a brief outlook.

5.2 Model Hamiltonian

The system dynamics we are interested in, is described by a generalized XY Hamiltonian of the form

$$H/\hbar = \sum_{i,j} \frac{J'(\theta_i, \theta_j)}{|\mathbf{r}_{ij}|^3} (S_i^+ S_j^- + h.c.) + \sum_j h_j S_j^z$$
(5.1)

where S_j^{α} are spin operators at the lattice site j, \mathbf{r}_{ij} is the distance vector between i and j, and the inter-qubit couplings $J_{ij} = \frac{J'(\theta_i, \theta_j)}{|\mathbf{r}_{ij}|^3}$ are in frequency units. The last term describes a disordered transverse field, which reflects the disorder in the microscopic qubit frequencies (i.e. Josephson energy).

The key element of our implementation is the realization of different patterns of quantum frustration [100] by tuning the form of the interaction couplings (Fig. 5.4). As discussed below, the latter display a rich dependence as a function of the dipole angles θ_j (see Fig. 5.4b): this dependence is essential to tune the coupling between different spins from positive to negative, or to (approximately) set it to 0. Even more crucially, the

 $1/|\mathbf{r}_{ij}|^3$ dependence (see Fig. 5.4a) can be exploited to further modify the magnitudes of the couplings. In the theory of 1D and ladder systems [101, 102], where the dipolar interaction is effectively local, this allows independent tunability of nearest-neighbor (NN) and next-nearest-neighbor (NNN) exchange, a fundamental ingredient to realize bond-order solids [31, 103]. We focus specifically on this case below, showing how, within our proposal, such states of matter are robust against both disorder and qubit decoherence. The setup can be straightforwardly extended to 2D geometries, where qualitatively new features emerge due to the non-locality of the dipolar couplings: in particular, the dynamics of Eq. (5.1) can by-pass the Mermin-Wagner-Hohenberg theorem [104, 105], and thus support phases of matter with true long-range order even at finite temperature [106]. We emphasize that all of these phenomena are directly accessible within our proposal thanks to the naturally occurring dipolar interactions, while such interactions would be challenging to be implemented via wiring.

5.3 Circuit QED implementation of dipolar XY models

We now describe how the many-body dynamics of Eq. (5.1) can be realized in our setup, using state-of-the-art circuit QED technology, with the design flexibility and long coherence times $1/\kappa$ ($\kappa \leq 2\pi \times 100$ kHz) of Transmon qubits [30, 95]. This is schematically illustrated in Fig. 5.1. Several Transmon qubits (in red) fabricated on a piece of sapphire (light blue area) which are mounted inside a waveguide cavity (grey box). The Transmon qubits can be fabricated in an essentially arbitrary lattice configuration with locally controllable orientation. The waveguide cavity around the qubits is used to readout the state of selected qubits and apply a drive to a subset of qubits, providing means for both adiabatic state preparation and probing (see next section). This selectivity can be again achieved by partially rotating the qubits inside the cavity by a few degrees, such that their dipole moment has a finite overlap with the electric field in the cavity. To add more flexibility to the setup, subsets of qubits can be fabricated on different pieces of sapphire. For simplicity these individual pieces are not shown in Fig. 5.1. Using a SQUID in the qubit circuit and a multi coil setup to change the flux for each qubit (as realized in [107]), one should be able to reduce the disorder in the transition frequencies to less than 1% (around $2\pi \times 30$ MHz spreading in h_i). This corresponds to a disorder

strength in the model Hamiltonian of order $\delta h \simeq 0.3$, relative to the exchange matrix elements $J_{ij} \simeq 2\pi * 100$ MHz.



Figure 5.1: Circuit QED implementation of spin models. (a) Drawing of one half of a waveguide cavity with two microwave coupler's. Inside the cavity is a piece of sapphire with multiple Transmon qubits arranged on a triangular lattice. The rotation angles of the qubits are chosen such, that only a few of them couple to the fundamental mode of the cavity with a predetermined coupling strength. (b) The cavity can be loaded with an arbitrary subset of qubits e.g. considering the ones in the dot-dashed box in (a) realizes a triangular ladder. Here, the bond thickness denotes the interaction strength of nearest-neighbor (J_1) and next-nearest-neighbor (J_2) interactions. Additional longer-range contributions are represented by dashed lines (only the strongest are shown). (c) Sketch of the dimerized phase of the extended XY model in Eq. (5.1). The phase can be understood as a solid of local triplet states, denoted by the shaded areas.

The unique feature of our approach is the design flexibility in the interqubit interaction which results from the dipole-antenna structure of the Transmon. As one can expect, two dipole antennas in the near field interact like two magnetic spins [108]. By designing the shape and size of the antenna we can realize large interaction strengths ($J_{ij} \approx$ $2\pi \times 100$ MHz) at inter-qubit distances of about 1 mm. The interaction between the qubits ultimately comes from an effective capacitance between the antennas, similar to [91, 92], with an angle and distance dependence akin to magnetic dipoles.

5.3.1 HFSS simulations

To confirm and quantitatively assess the inter-qubit dipolar interactions, we have performed a finite element study to determine the coupling strength and dependence on distance and angle. These simulations are carried out by *ab initio* numerically solving the 3D Maxwell equations on finite grids (HFSS software [109]), and as such provide an extremely accurate quantitative benchmark for our modeled inter-qubit couplings. The quantitative insights gathered from the simulations are of basic importance to underpin the effects of the specific configuration that would be accessible in realistic setups, where the influence of the surrounding waveguide cavity and finite size effects of the Transmon antenna have to be systematically understood. It should be noted that finiteelement calculations of a structure with objects that are orders of magnitude different in dimensions requires plenty of computational resources.



Figure 5.2: Mesh used to calculate the electromagnetic fields around the qubits. The top left half of the image shows the meshing needed for the cavity that contains a sapphire substrate with two qubits on it. The top right half shows meshing around one of the qubits. The coordinate system used in this paper is defined in the lower left.

In this work, extremely fine meshing is needed in areas where accurate understanding of the physics of small objects (like qubits) is needed. Fig. 5.2 shows how different mesh sizes are used for different structure size to accurately calculate the eigenmodes of the electromagnetic fields of two qubits in a cavity. Typical cavity dimensions for all simulations are: width= 2.5 cm, depth=3 cm, height=1 cm. In the following, the coupling strength is defined as the minimum difference in the mode frequencies of two qubits when the inductance of one qubit is swept in order to tune its mode frequency in and out of resonances with the other(see Fig. 5.9). All simulations regarding the coupling strength of the Transmon qubits were done with typical Transmon parameters i.e. $E_C \approx 250$ MHz, $\sqrt{8E_jE_c} \approx 6$ GHz and $E_j/E_c \geq 50$. The antenna height used for all simulations, unless otherwise specified, was 1 mm.

5.3.1.1 Single qubit inside cavity



Figure 5.3: a) Extracted capacitance of the qubit as a function of the width of the antenna paddles. The dashed line is a linear fit to the data. b) Qubit-Cavity coupling strength g as a function of its position inside the cavity. The qubit was placed in the center of the cavity in the y and z direction and was moved along the x axis. The dashed line is a cosine fit to the data. c) Coupling g of the qubit to the cavity as a function of its antenna length. Since a change in antenna length will also change the resonance frequency, the width of the antenna has been adjusted to keep the resonance frequency constant. The dashed line is a linear fit to the data.

In order to better understand the effect of the antenna geometry on the qubit properties, we start our analysis form the single qubit case. First, the width of the antenna pads w was changed and the corresponding capacitance was extracted as shown in Fig. 5.3 a). It can be concluded that the capacitance increases linearly as a function of the pad width.

Then, the coupling strength of the qubit to the cavity J was calculated as a function of position of the qubit with respect to the side wall of the cavity. The coupling strength has a cosine form as expected from a standing wave in the cavity. The results are shown in Fig. 5.3 b).

The coupling strength J was also studied as a function of the pad height h as shown in Fig. 5.3 c) where the qubit was located at the center of the cavity. It can be seen from the figure that the coupling strength increases linearly as a function of the antenna height. This is in agreement with dipole antenna physics.

5.3.1.2 Coupling strengths from HFSS: Two-qubit case

A set of results can be seen in Fig. 5.4. Assuming a dipole-dipole interaction with an additional component due to the dispersive coupling of both qubits to the cavity with strength g (determined from independent simulations), we expect the following spatial dependence:

$$J_{1,2} = \frac{g^2 d_m^2}{2\Delta} \sin \phi_1 \sin \phi_2 + - J_0 d_m^2 \frac{\sin \theta - 3 \sin \theta_1 \sin \theta_2}{(r - r_m)^3}.$$
 (5.2)

The first term takes into account the qubit-qubit interaction mediated via the fundamental cavity mode with ϕ_1, ϕ_2 the orientation of the qubits relative to the orientation of the electric field in the cavity. Higher order cavity modes contribute with a similar term with a coupling g' depending on the mode structure; as these are typically very far detuned, we neglect them in the following. The second term is the direct dipole-dipole interaction between the qubits with $r = |\mathbf{r}_{12}|$ being the distance between their centers. Both, the qubit-cavity interaction strength (which depends on the location of the qubit in the cavity) and the dipole-dipole interaction J_0 get modified by the length of the antenna, giving rise to the term d_m in Eq. 5.2, with d_m the normalized antenna length. The remaining variables in this equation are the qubit cavity detuning $\Delta \approx 2\pi \times 1.5$ GHz and the angles θ_1, θ_2 with $\theta_- = \theta_1 - \theta_2$. These are the angles formed by the two dipoles with respect to a line connecting their centers, as can be seen in Fig. 5.4(b). r_m corrects for finite size effects of the dipole antennas.

One can see in Fig. 5.4(a) that the qubit-qubit interaction strength in our numerical simulations very closely follows the analytical expression Eq. 5.2. The only fitting parameters in this expression are J_0 , which is the same for all three datasets, and the r_m , which is adjusted for each length. The dependence on the distance for different antenna sizes can be seen in Fig. 5.4(a). The interaction strength between the qubits for a parallel orientation is negative for short distances and falls of as $1/r^3$. For larger distances it becomes positive due to the additional coupling mediated off-resonantly via the cavity. This leads to the effect that qubits at a given distance (in the case of our cavity about 3.5 mm) do not interact with each other as these terms exactly cancel out.



Figure 5.4: Microscopic finite-element simulation (see text) of the dependence of the coupling $J_{i,j}$ between two qubits. Panel (a): distance dependence at $\theta_i = \theta_j = 90^\circ$ for three different antenna length d_m (given in mm). The dashed line indicates $J_{i,j} = 0$. The inset displays a typical result from HFSS simulations [109], with the dipoles surrounded by the computed electric field (intensity decreasing from red to blue). Panel (b): angular dependence illustrated for the case $\theta_i = \theta_j = \theta$ at distance r = 1.5 mm. The solid line plots Eq. 5.2 with J_0, r_m extracted from best fits in (a).

Such a canceling of the direct inter-qubit interaction can be very useful for quantum information experiments.

By rotating one qubit around the other, as shown in Fig. 5.4(b), we can see that the interaction strength goes from a negative value for a parallel orientation to a twice as large positive value for collinear qubits, as expected for a dipole-dipole interaction. The analytical curves agree well with the numerical simulations and demonstrate that the spatial dependence of the interaction behaves like a magnetic dipole-dipole interaction. Furthermore we note that the cavity mediated term can be fully suppressed by rotating the qubits perpendicular to the electric field of the cavity mode, as shown schematically in Fig. 5.1. The finite-element simulations are in very good agreement with a simple analytical circuit model described in the next section, which already captures the important interaction features.

5.3.2 Additional simulations for two qubit

Further understanding more insight beyond paper.

5.3.2.1 Moving one qubit along Y-axis

Two qubits are placed on a piece of sapphire separated by a distance of 1.5 mm. One of the qubit is swept along the Z-axis, at a distance of 1 mm center to center distance between qubits the interaction strength coupling 'J' is zero. At a distance of 1 mm, the angle between the qubits approaches to 35° . From classical dipolar physics, when the two dipole antennas are at an angle of 35° the coupling strength between the antennas is zero [56].



Figure 5.5: The qubits are placed symmetric to the cavity and one of the qubit is swept along the y-axis. Coupling 'J' as function of distance between the two qubits along Y-axis 'd' for a fixed distance along Z-axis.

5.3.2.2 Coupling strength as function of distance between two qubits back to back on two sapphire chips

In this simulations, two qubits are placed in a cavity on two separate sapphire chips. The distance d_{BB} is varied to understand the qubit-qubit interaction with a vacuum gap in between the sapphire chips. The interaction strength between the qubits goes from a negative value to a positive value. As the distance of 1 mm between the qubits the coupling strength is zero, the reason for this zero is that interaction strength goes from a negative value to positive value and cavity mediated destructive and direct interference. By further increasing the distance (d_{BB}) the coupling strength between the qubits becomes rapidly weaker. The maximum coupling strength 'J' is obtained when the two sapphire chips are touching each other. In this scenario, the coupling strength

between the qubit's is mainly influenced by the sapphire chips. Sapphire has a dielectric constant of $\epsilon_r = 11$, hence influencing the capacitive interaction between the qubit's.



Figure 5.6: Coupling strength 'J' as function of distance between two qubits mounted back to back d_{BB} . In the illustration transmon qubits are placed on two sapphire chips of about 330 μm thickness. The coupling strength J reduces quickly to 10's of MHz because of the increased distance between the sapphire substrates.

5.3.3 Changing the coupling in-situ

To add more flexibility to the setup, each row of qubits can be fabricated on an individual piece of sapphire. These two pieces can then be moved relative to each other (e.g. along the z-direction) via a piezo actuated stage (as e.g. used in [110, 111]) which allows for an in-situ change of the coupling ratio J_2/J_1 . As the distance for all qubits in one row does not change J_2 stays constant. In contrast J_1 will vary with the distance of the two sapphire pieces. How the ratio of J_2/J_1 changes when the sapphire pieces are moved apart along the , can be seen in Fig. 5.7.

One can see that changing the distance by about 100 μ m changes the coupling ratio by about a factor of two. Typical piezo actuators allow a positioning to much better than 1 μ m over distances of a few millimetres, making this approach technically challenging but feasible. In order to be able to move the sapphire pieces, the sapphire has to pass through the cavity wall via thin slit. This thin slit will be a waveguide below cut off and poses no problem for the desired (rather low) quality factor of the cavity. Once outside the superconducting wall, the sapphire can be clamped to the piezo stage.

The two sapphire pieces can also be moved along the y-direction, which is perpendicular to the illustration in Fig. 5.7. This would again change the coupling between the rows
as illustrated in Fig.2 in the main text. For feasible parameters a change of about a millimeter, would be necessary to change the coupling ratios from about 0.25 to 1.



Figure 5.7: Coupling J_1 and J_2 and the ratio J_2/J_1 as a function of distance S between qubit rows. This simulation was run with a 1 mm antenna. Each row of qubits in the ladder model can be fabricated on an individual piece of sapphire. These two pieces can then be moved relative to each other via a piezo actuated stage which allows for an in-situ change of the coupling ratio J_2/J_1 . As the distance for all qubits in one row does not change J_2 stays constant. In contrast J_1 will vary with the distance of the two sapphire pieces. The inset shows a side view of the two sapphire pieces with the two rows of qubits (red blocks) on top. The two rows are offset from each other on the two sapphires. This offset can be adjusted to match the desired couplings for S approaching zero.

5.3.4 Qubit state measurement and Flux tuning

An advantage of a circuit QED setup is the ability to measure the state of a predetermined but otherwise arbitrary subset of qubits in the lattice. A measurement of $\langle S_i^z \rangle$ can be realised in circuit QED by probing the cavity transmission in the limit of a dispersive qubit cavity coupling. With the newly developed quantum limited amplifier technology it is nowadays possible to detect quantum jumps of an individual qubit [33, 112] and the parity of a two qubit state [87]. Using this measurement techniques one has to take into account the strong interaction between the qubits, which means that a direct measurement of $\langle S_i^z \rangle$ in the uncoupled basis is not possible [113]. We would rather project This effect can be avoided by effectively switching off the coupling by detuning a subset of qubits from each other and all other qubits by more than J in a time faster than 1/Jin order to be non adiabatic. This detuning can be achieved by realizing the qubits one wants to measure with a SQUID instead of a single Josepshon junction. The challenge in the 3D architecture is, to realize a fast tune-ability of the flux through the SQUID. Some of the problems that typically appear are a slow response time of the flux bias circuit and a possibility of an oscillation of the flux due to ringing of the circuit. These led to the development of fast flux bias lines and several methods to eliminate the response of the line by deconvolution methods solved by using a magnetic hose [114, 115].

In our case the requirements on the flux bias lines are reduced. We do not care about a small uncertainty in the qubit frequency after the flux change as additional dephasing won't be relevant for the measurement of $\langle S_i^z \rangle$. If ringing of the circuit proves to be a problem, one could use an asymmetric SQUID design for the Transmon qubit [116] and use both flux sweet spots to reduce flux sensitivity. Furthermore, we do not care about addressability as only a subset of qubits will be sensitive to the flux. Also we only need two flux configurations: (1) all qubits in resonance and (2) the qubits that have to be measured sufficiently detuned from the rest.

Combining these relaxed requirements should allow us to adjust the flux with the help of small coils (50-100 nH) just outside the cavity made out of copper hose [115]. Simulations and preliminary room temperature experiments show that a current of \approx mA is sufficiently large to generate the desired flux. Changing the current in one of these coils will effect more than one qubit, though with different magnitude. Adjusting the currents in different closely spaced coils using e.g. an optimal control algorithm should in principle allow us to generate the desired flux values for our application at the qubit location even though there are inter-dependencies. The switching times will be limited by Eddy currents in the cavity wall. These can be reduced by implementing the flux bias lines inside the cavity with appropriate filtering to avoid an undesired coupling of the cavity or qubit to the environment. Another method to measure correlations functions of interacting qubits coupled to a cavity was recently published in a very similar context [117]. In this paper the authors show that the spectrum of the resonator is directly related to the correlation function of the coupling operator between the resonator and the interacting qubits. This method can be adapted in the context of our proposal and be used to readout correlation functions. Even more information can be extracted by making use of multiple resonator modes with different coupling to the qubits. The advantage here is, that no fast switching of the magnetic bias field is necessary.

5.4 Circuit model

In this section, details of a circuit model that represents two-qubits inside a cavity are explained. The model is used to evaluate the coupling strength between the elements, and support the HFSS simulations.

5.4.1 Circuit diagram

Dissipation-less LC resonators are used to model the cavity and qubits (see also [118] or [119] for a refined method including dissipation). The nonlinearity of the qubits is not taken into account as it does not affect the coupling strength. Capacitors are used in order to account for the coupling between elements. The coupling between each qubit and the cavity is represented by a coupling capacitor C_{qcav} . As qubits are assumed to couple to each other through their dipole antenna, a total of four capacitors are used to represent coupling of each antenna pad to the two pads of the other qubit (see Fig. 5.8).

5.4.2 Coupling strength

In order to calculate the coupling between elements, the following method is used: The impedance of the system (Z_{in}) seen through one of the qubits (here Q_1) is calculated. The poles of the impedance are the resonance modes of the total system, in this example three. By sweeping the inductance L_1 (of Q_1) we can see the avoided crossing between the modes (see Fig. 5.9). Coupling is defined as the minimum distance between the two modes at the avoided crossing.



Figure 5.8: a) Circuit model corresponding to two qubits in a cavity. Values of the four capacitors drawn in red decay as $1/r^2$ where r is the center to center distance between pads of the antenna. This models the dipole-dipole coupling of two qubits. The two purple structures on Q1 and Q2 represent the physical shapes of the qubits. b) Capacitance between two co-planar paddles (solid line) as a function of the separation s according to equation 5.3. The corresponding structure is shown at the bottom. The dashed line shows the quadratic decay used in this study to estimate the coupling capacitance. The structure at top right shows which capacitors in the circuit model follow the quadratic decay.

A further step was taken by relating the inter-qubit coupling capacitances (as in Fig. 5.8 a)) to the physical distances among the qubit antenna poles. This distance dependence is assumed to be of the following form [120]:

$$C_{ab} = (2\epsilon/\pi) \ln \frac{(a_L - b_L)(a_R - b_R)}{(a_R - b_L)(a_L - b_R)}$$

= $(2\epsilon/\pi) \ln \frac{(w+s)^2}{(w+s)^2 - w^2}$ (5.3)

where ϵ is the average over dielectric constants of the substrate and the air (or vacuum), $w = w_a = w_b$ is the width of antenna paddles and s is the separation between closest edges as shown in the inset of Fig. 5.8 b). The parameters a_L, a_R, b_R, b_L are defined in Fig. 5.8 b).

As this study concerns square shaped paddles (see top right inset of Fig. 5.8b)), it is not possible to apply equation 5.3 to any arbitrary relative position of the two qubits without complicated corrections. Therefore, just to demonstrate the general behavior, a simplified relation (Eq. 5.4 follows Eq. 5.3 when the separation s becomes much larger than the paddle width w - this effect is shown in Fig. 5.8 b)) for the coupling is chosen which assumes that the capacitance decays quadratically as a function of the distance:

$$C_{ab} = \frac{a}{s^2} \tag{5.4}$$

where a is a coefficient used to adjust the coupling strength.

This means that the coupling strength $J_{i,j}$ can be calculated based on the physical position of the two qubits with respect to each other. In this study, Q_1 is assumed to be fixed in position and Q_2 changes its distance and angle with respect to Q_1 .



Figure 5.9: Example for three modes of a system with two qubits and a cavity. The minimum distance of two modes is defined as the coupling $2J_{i,j}$ if it is two qubits and as 2g if it is the qubit cavity coupling. Note that despite the fact that Q_2 has 9 nH of inductance, the crossing happens at lower inductances due to the coupling to Q_1 . Dashed line shows how resonance frequency of a typical LC circuit would behave without coupling to the other resonators.

Assuming that the two qubits are placed parallel with respect to each other, we have calculated the coupling $J_{i,j}$ as a function of the center-to-center distance. In Fig. 5.10 the results are shown for two qubits without a mediating cavity. As can be seen, the coupling is heavily suppressed along a straight line. This line can be thought of as when the dipole electric field from Q_1 is horizontal, that is perpendicular to the dipole antenna of Q_2 and hence, there will be no coupling. This is typical behavior of dipoles and is well expected. However, by adding a cavity to the model (see Fig. 5.10 b)), one can observe that the straight line shape of suppression will no more hold and instead, a curved line emerges.

Adding the cavity has certain consequences on how two qubits couple. For example, if one moves Q_2 along the x-axis while it is held at y = 1 mm (or y = -1 mm), the coupling might be suppressed twice. This is verified by HFSS simulations as seen in Fig. 5.4. Our



Figure 5.10: a) Contour plot showing the coupling strength between two qubits $J_{i,j}$ as a function of the distance between them, in the x and y direction, in absence of the cavity. The color scale is logarithmic to highlight the rapid change in $J_{i,j}$. In such a case, the dipole physics dominates and the coupling will be suppressed along a straight line. The white area is not accessible due to the dimensions of the qubits. b) Contour plot showing the coupling strength between two qubits $J_{i,j}$ as a function of the distance between them, in the x and y direction, in presence of the cavity. The color scale is logarithmic to highlight the rapid change in $J_{i,j}$. The cavity distorts the straight line of suppressed coupling opening new possibilities for engineering of the coupling between two qubits. The white area is not accessible due to the dimensions of the qubits.

lumped-element model shows behavior that matches the HFSS simulations, as can be seen in Fig. 5.11.

5.5 Hamiltonian derivation

In this section, I briefly review the derivation of the microscopic spin Hamiltonian between interacting Transmons.

5.5.1 Interacting Transmons

Let's consider two interacting transmons as depicted in Fig. 5.12. The Lagrangian describing this system is given by



Figure 5.11: Coupling J as a function of distance along x axis. The center to center separation along the Y axis is kept at 1 mm and one qubits is moved along the x axis with respect to the other. The results suggest a double suppression of the coupling in agreement with the circuit model (dashed curve).



Figure 5.12: Lumped element circuit model for two capacitively coupled superconducting qubits.

$$L = \sum_{i=1,2} \frac{1}{2} C_i \dot{\Phi}_i^2 + \frac{1}{2} C_Q (\dot{\Phi}_1 - \dot{\Phi}_2)^2 + \sum_{i=1,2} E_J^{(i)} \cos\left(\frac{\Phi_i}{\Phi_0}\right)$$
(5.5)

with $\Phi_0 \equiv \hbar/(2e)$. The Hamiltonian, $H = \sum_{i=1,2} Q_i \dot{\Phi}_i - L$ can then be written as

$$H = \frac{1}{2}\vec{Q}_T C^{-1}\vec{Q} - \sum_{i=1,2} E_J^{(i)} \cos\left(\frac{\Phi_i}{\Phi_0}\right),$$
(5.6)

with $\vec{\Phi}_T \equiv (\Phi_1, \Phi_2)$. For small phase fluctuations, I can expand the cosine and get $H = H_0 + H_1$, with

$$H_0 = \frac{1}{2}\vec{Q}_T \mathcal{C}^{-1}\vec{Q} + \frac{1}{2}\vec{\Phi}_T \mathcal{L}^{-1}\vec{\Phi}$$
(5.7)

$$H_1 \approx -\sum_{i=1,2} \frac{E_J^{(i)}}{24\Phi_0^4} \Phi_i^4,$$
 (5.8)

where

$$\mathcal{C} = \begin{pmatrix} C_1 + C_Q & -C_Q \\ -C_Q & C_2 + C_Q \end{pmatrix},$$

$$\mathcal{C}^{-1} = \frac{1}{D} \begin{pmatrix} C_2 + C_Q & C_Q \\ C_Q & C_1 + C_Q \end{pmatrix},$$

$$\mathcal{L}^{-1} = \begin{pmatrix} 1/\alpha_1 & 0 \\ 0 & 1/\alpha_2 \end{pmatrix},$$

 $D \equiv C_1 C_2 + C_1 C_Q + C_2 C_Q$ and $1/\alpha_i \equiv E_J^{(i)}/\Phi_0^2$. Now by quantizing this Hamiltonian using the usual ladder operator replacement for Q_j and Φ_j which is

$$Q_{j} = iQ_{j}^{\text{ZPF}}(a_{j}^{\dagger} - a_{j}), \Phi_{j} = \Phi_{j}^{\text{ZPF}}(a_{j}^{\dagger} + a_{j}),$$
(5.9)

where $Q_j^{\text{ZPF}} \equiv \sqrt{\hbar\omega_j C_j^{\text{eff}}/2}, \ \Phi_j^{\text{ZPF}} \equiv \sqrt{\hbar\omega_j \alpha_j/2}, \ C_1^{\text{eff}} \equiv \frac{D}{C_2 + C_Q}, \ C_2^{\text{eff}} \equiv \frac{D}{C_1 + C_Q}.$ Here, $\omega_1, \ \omega_2$ are the two solutions of $|\mathcal{L}^{-1} - \omega^2 \mathcal{C}| = 0$. This quantization gives

$$H \approx \sum_{i=1,2} \omega_i n_i - \sum_{i=1,2} \Lambda_i n_i^2 + \lambda_{12} (a_1^{\dagger} a_2 + \text{h.c.}), \qquad (5.10)$$

where $\Lambda_j \equiv \frac{E_j^{(j)}}{4} \left(\frac{\Phi_j^{\text{ZPF}}}{\Phi_0}\right)^4$, $\lambda_{12} \equiv \frac{C_Q}{D} Q_1^{\text{ZPF}} Q_2^{\text{ZPF}}$, and $n_j \equiv a_j^{\dagger} a_j$. Notice that this Hamiltonian has been written in a basis of local modes. Diagonalization of the quadratic

part via a Bogoliubov transformation gives rise to a Hamiltonian of the form [118]

$$H \approx \sum_{i=1,2} \xi_i n_i - \sum_{i=1,2} \alpha_i n_i^2 - \chi_{12} n_1 n_2.$$
(5.11)

5.5.2 Interacting transmons inside a cavity



Figure 5.13: Lumped element circuit for two interacting superconducting qubits in a cavity resonator.

Let's now consider the circuit depicted in Fig. 5.13 which is a simplified version of the circuit shown in Fig. 5.8 a) with the capacitive coupling network between the qubits reduced to one capacitor. This transformation can be easily done for a fixed qubit-qubit distance. The simple lumped element model will provide the right Hamiltonian but does not capture the angle and distance dependence explicitly. The Lagrangian of the system is given as

$$L = \sum_{i=1,2} \frac{1}{2} C_i \dot{\Phi}_i^2 + \sum_{i=1,2} E_J^{(i)} \cos\left(\frac{\Phi_i}{\Phi_0}\right)$$

$$+ \frac{1}{2} C \dot{\Phi}_R^2 - \frac{\Phi_R^2}{2L} + \frac{1}{2} C_Q (\dot{\Phi}_1 - \dot{\Phi}_2)^2$$

$$+ \sum_{i=1,2} \frac{1}{2} C_0 (\dot{\Phi}_i - \dot{\Phi}_R)^2.$$
(5.12)

The associated Hamiltonian $H = H_0 + H_1$ can be written as equations (5.7) and (5.8), with

$$\mathcal{C} = \begin{pmatrix} C_1 + C_Q + C_0 & -C_Q & -C_0 \\ -C_Q & C_2 + C_Q + C_0 & -C_0 \\ -C_0 & -C_0 & C + C_0 \end{pmatrix},$$
$$\mathcal{L}^{-1} = \begin{pmatrix} 1/L_1 & 0 & 0 \\ 0 & 1/L_2 & 0 \\ 0 & 0 & 1/L \end{pmatrix}.$$

Here, $L_i \equiv \frac{\Phi_0^2}{E_J^{(i)}}$. Quantizing this Hamiltonian gives

$$H \approx \sum_{i=1,2,R} \omega_i n_i - \sum_{i=1,2} \Lambda_i n_i^2 +$$

$$+ \sum_{i \neq j=1,2,R} \lambda_{ij} (a_i^{\dagger} a_j + \text{h.c.}),$$
(5.13)

with similar expressions for the coupling constants as in Eq. (5.10)¹. Here, the qubit and cavity are far detuned. However, the qubit-cavity coupling may be sufficiently large to mediate virtual transitions between both qubits through the cavity. This effect will be relevant when $\frac{\lambda_{iR}\lambda_{jR}}{\Delta_j} \sim \lambda_{12}$, where $\Delta_j \equiv -\omega_j + \omega_R$. Performing a canonical transformation

$$H \to e^S H e^{-S},\tag{5.14}$$

with

$$S \equiv \sum_{i=1,2} \varepsilon_{iR} (a_i^{\dagger} a_R - a_R^{\dagger} a_i), \qquad (5.15)$$

and taking $\varepsilon_{iR} \approx -\frac{\lambda_{iR}}{\Delta_i}$, and obtain to second order in the transformation, an effective Hamiltonian (in a rotating frame with respect to the resonator)

$$H_{\text{eff}} \approx \sum_{i=1,2} \tilde{\Delta}_i n_i - \sum_{i=1,2} \tilde{\Lambda}_i n_i^2 +$$

$$+ \sum_{i \neq j=1,2} J_{ij} (a_i^{\dagger} a_j + \text{h.c.}),$$
(5.16)

¹For the qubit-cavity coupling the capacitance C_Q is substituted by C_0 in such expressions.

with the renormalized constants are related to the original energies as

$$\tilde{\Delta}_i = -\Delta_i (1 - \varepsilon_{iR}^2), \qquad (5.17)$$

$$\tilde{\Lambda}_i = \Lambda_i (1 + 4\varepsilon_{iR}^2), \qquad (5.18)$$

$$J_{ij} = \lambda_{ij} + \frac{\lambda_{iR}\lambda_{jR}}{2\Delta_j}, \qquad (5.19)$$

where i = 1, 2. The Hamiltonian (5.16) can be readily generalized for N qubits by letting i = 1, ..., N, and conveniently diagonalized into the form written in Eq. (5.11). Also, it can be written in a spin language, by considering sufficiently large qubit anharmonicities. Typical anharmonicities for Transmon qubits in the range of 200 - 300 MHz are sufficient to realize a successful state preparation, while still staying in the Transmon limit $(E_J/E_C > 40)$. This allows us to replace the Bosonic operators by spin 1/2 operators, so the effective Hamiltonian for two capacitively coupled superconducting Transmon qubits inside a cavity is given by

$$H_{\text{eff}} \approx \sum_{i} \omega_i S_i^z + \sum_{i \neq j} J_{ij} (S_i^+ S_j^- + \text{h.c.}).$$
(5.20)

Note that the angle and distance dependence of the qubit coupling is hidden in the J_{ij} in this derivation.

5.6 Conclusions

From the finite element numerical simulations, we show how to use the naturally occurring dipolar interactions in 3D superconducting circuits to realize a platform for analogue quantum simulation of XY spin models. The possibility of realizing arbitrary lattice geometries with locally-tunable dipole moments [29], in combination with their large interaction strength, opens the door to the investigation of a series of phenomena in quantum magnetism in both 1D [121, 122] and 2D [106], complementing the remarkable developments in cold atom and trapped ion systems [96, 97, 98]. The idea discussed in this chapter [29] are not limited to Transmon qubits, but could be implemented with, e.g., Xmon qubits [89] or Fluxonium qubits coupled to an antenna [123]. It would be interesting to explore these developments in view of realizing Hamiltonian dynamics for surface code architectures [8] or as a building block for coupled cavity array experiments [124, 125].

Chapter 6

Publication 2: Characterization of low loss microstrip resonators as a building block for circuit QED in a 3D waveguide

My contribution

I have contributed in designing the High-Q microstrip resonators and writing the manuscript. The numerical simulations for designing the MSR's and characterization of the devices is initially carried out by me. In collaboration with other authors, the final characterization results are discussed in this paper. Here we present the microwave characterization of microstrip resonators made from aluminum and niobium inside a 3D microwave waveguide. In the low temperature, low power limit internal quality factors of up to one million were reached. We found a good agreement to models predicting conductive losses and losses to two level systems for increasing temperature. The setup presented here is appealing for testing materials and structures, as it is free of wire bonds and offers a well controlled microwave environment. In combination with transmon qubits, these resonators serve as a building block for a novel circuit QED architecture inside a rectangular waveguide.

6.1 Motivation

M icrowave resonators are an important building block for circuit QED systems where they are e.g. used for qubit readout [126, 127], to mediate coupling [128] and for parametric amplifiers [34]. All of these applications require low intrinsic losses at low temperatures $(k_BT \ll hf_r)$ and at single photon drive strength. In this low energy regime, the intrinsic quality factor, which quantifies internal losses, is often limited by dissipation due to two level systems (TLS) [129, 130]. These defects exist mainly in metal-air, metalsubstrate and substrate-air interfaces as well as in bulk dielectrics [130, 131, 132, 133]. Two common approaches exist, to improve the intrinsic quality factor of resonators. Either one reduces the sensitivity to these loss mechanisms by reducing the participation ratio [130, 59, 134] or tries to improve the interfaces by a sophisticated fabrication process [135, 136]. Reducing the participation ratio requires to reduce the electric field strength. This is typically done by increasing the size of the resonator [132] or even implementing the resonator using three dimensional structures [59].

6.2 Design of a MSR in a waveguide

Our approach, a microstrip resonator (MSR) in a rectangular waveguide (Fig. 6.1), combines the advantages of three dimensional structures with a compact, planar design [127, 30]. The sensitivity to interfaces is reduced, since the majority of the field is spread out over the waveguide, effectively reducing the participation ratio [134]. Another advantage is, that the waveguide represents a clean and well controlled microwave

environment [56] without lossy-seams [137] close to the MSR. As the MSR is capacitively coupled to the waveguide, no wirebonds [138] or airbridges [139] are required, which can lead to dissipation or crosstalk.

The U-shaped MSR (Fig. 6.1 a)) is a capacitively shunted $\lambda/2$ resonator. Due to the open end, a voltage maximum occurs at the ends and a current maximum in the center [56]. Considering that the MSR is patterned on silicon, the resonance frequency of the fundamental mode is expected at 11.3 GHz. The additional shunt capacitance between the legs shifts the resonance frequency down to about 8 GHz.



Figure 6.1: MSR layout. a) Sketch of the MSR on a substrate. b) Sketch of the cross section of the waveguide with MSR inside. The dashed line indicates the electric field strength of the fundamental mode inside the waveguide.

The coupling of MSR, depends on the position of the MSR in the waveguide along the x-axis. A critically coupled setup is inevitable to get trustworthy results of Q_{int} and Q_c , in particular in the single photon limit. To accomplish such a setup, simulations were performed.

6.2.1 Finite element simulations on the coupling

To estimate the coupling between the MSR and the waveguide, numerical simulations using finite element solver was performed.

Figure 6.2 illustrates the two considered cases. At first, the MSR was swept from the center towards the wall. Fig. 6.3 a) shows the results of the coupling. A critically coupled setup in the single photon limit requires a coupling quality factor on the order of 1×10^5 to 1×10^6 , depending on the measured MSR. In the available waveguides, there are only discrete slots to place the sample. The first off-centered slot is around



Figure 6.2: Sketch of the simulated setups. a) In simulations the MSR was shifted from the center towards the wall. b) MSR placed in the center with an additional sapphire substrate next to it, which displaces the field inside the waveguide. This leads to an asymmetry of the electric field over the centrally placed MSR, thus a non vanishing coupling. In simulations the substrate was shifted from the MSR towards the wall.



Figure 6.3: Simulation results of the coupling quality factor for different positions of the MSR in the waveguide. a) MSR shifted from the center to the waveguide wall (illustrated in Fig. 6.2 a)). b) MSR placed in the center and the empty substrate being shifted away towards the wall (Fig. 6.2 b)).

3 mm from the center, which leads to a coupling quality factor between 1×10^3 and 1×10^4 , being around two magnitudes below critically coupled.

To displace the electric field in the waveguide, the MSR is placed in the center and an empty sapphire substrate in a neighbouring slot, due to the higher ϵ_r of sapphire. This is illustrated in Fig. 6.2 b). From the simulations with the MSR placed in the center and the empty substrate being shifted towards the wall (Fig. 6.3 b)). The substrate being one slot off center (neighbouring the MSR) leads to a coupling of around 1×10^5 . For two slots off center the desired coupling quality factor reaches around 1×10^6 . It is important to note, that for such high coupling quality factors, effects like the MSR having a slightly asymmetric leg length, or being placed off center on the chip or placed entirely off center, can have a big impact on the coupling. For instance simulations showed, that a displacement of 0.2 mm off-center, can lead to a factor of 4 in the coupling quality factor.

6.3 MSR and aluminium waveguide in detail

In this section, the photographs of the MSR on the silicon substrate, the MSR in the waveguide and the completely assembled waveguide are shown.



Figure 6.4: a) Photograph of a MSR with uneven leg length. b) Photograph of MSR placed inside the rectangular waveguide. Illustration, including dimensions of the waveguide and the MSR in Fig. 6.2. c) Mounting process. The MSR is slid in from the top. Two metal rods are used as guidance.

Figure 6.4 shows a photograph of the MSR a) and the MSR in the waveguide b). In c) the process of mounting the MSR is shown. The MSR is assembled to a holder and slid from the top into the waveguide.



Figure 6.5: Photograph of totally assembled waveguide. The waveguide used for the transmission measurement consist of three parts. In the middle section, three rows of samples can be mounted. In this picture only the middle row is used for a sample.

Fig. 6.5 shows the fully assembled waveguide. The waveguide consists of three parts. At each end there is an identical coupler to receive and launch the microwave signals.

The central part contains the samples. In this design no seams are present near the samples. It is possible to probe three samples, each in one of the three slots, which can be easily extended to more samples by using a different central section. One can see the individual slots for each sample, customized to the dimensions of our sample. In contrast to this waveguide, the copper waveguide only allows to probe a single sample at a time.

For microwave transmission measurements, the MSR is placed in a rectangular waveguide (Fig. 6.1 b)). The fundamental TE_{10} mode, which has electric field components only along the z-axis, is the sole propagating mode at the resonance frequency of the MSR. Its field strength varies along the x-axis with a maximum in the center [56] (dashed line in Fig. 6.1 b)). For the MSR placed off-center, the field strength is different on both legs, which leads to a capacitive coupling to the waveguide. Placed in the exact center of the waveguide, the field strength is equal on both legs of the MSR and the coupling vanishes. The coupling of the MSR in the waveguide, can also be varied with legs of different length. To accurately predict the interaction of the MSR with the waveguide numerical simulations is performed using a finite element solver.

MSRs are characterized in waveguides fabricated from oxygen free copper or aluminum. The waveguides were mounted to the baseplate of a dilution refrigerator and cooled down to 20 mK. The MSRs were analyzed regarding their resonance frequencies and quality factors by measuring S_{21} . The measured data is fitted using a circle fit routine [140] which utilizes the complex nature of the S-parameter.

6.4 Measurement results

To assess the performance of different materials, aluminum and niobium MSRs is investigated. The samples were fabricated using standard optical lithography techniques and sputter deposition of the metallic films. Structuring of the metal layer was done using a wet etching process for the aluminum samples and a reactive ion etching (RIE) process for niobium. After completely removing the photoresist, both samples were cleaned in an oxygen plasma.

Two sets of measurements were performed. First, the MSRs are measured under variation of input powers, ranging from below the single photon limit to several million photons circulating in the resonator. Second, to understand the TLS effects, the base temperature is ramped up to 1 K and performed measurements at single photon powers.

6.4.1 Internal quality factor dependence on circulating photon number and temperature

Fig. 6.6 a) shows the dependence of the internal quality factor on the circulating photon number in the MSR. All measurements show a clear trend of an increasing quality factor with the number of photons. This indicates that the MSRs are limited by TLS losses, as they get saturated with increasing drive powers [129]. From the measurements the highest single photon internal quality factor of one million for the two niobium MSRs placed in the aluminum waveguide. For high powers a Q_{int} of more than 8 million is measured. Other experiments, using a more sophisticated fabrication process, report similar internal quality factors for planar NbTiN resonators on deep etched silicon [135] or for planar aluminum resonators on sapphire [136]. Similar methods and materials might allow us to increase the single photon quality factor of the MSR.



Figure 6.6: Dependence of the internal quality factor on a) the circulation photon number in the MSR and b) the temperature (Al MSR). \diamond Nb MSR in Cu waveguide. • \diamond Nb MSR in Al waveguide. \checkmark Al MSR in Al waveguide. \blacktriangle Al MSR in Cu waveguide. a) All but one MSR/waveguide combinations show a clear increase of the internal quality factor with increasing photon number. The only exception is the MSR in the copper waveguide which seems to be limited by other losses. Both Nb MSRs in the aluminum waveguide show an internal quality factor of one million at the single photon limit. The measurements were taken at 20 mK. b) Internal quality factor of Al MSR in dependence of temperature, measured with single photon powers. The data are fitted to a model (dashed lines) combining decreasing TLS related losses (Eq. 6.2) and losses due to an increasing surface resistance with temperature (Eq. 6.1)

The trend of increasing Q_{int} is weakest for the aluminum MSR in the copper waveguide, which indicates that this MSR is not limited by TLS. Due to the normal conducting copper waveguide does not shield external fields. Thus vortices might limit the performance of the aluminum MSR in the copper waveguide [141]. This effect is not observed for the niobium MSR, due to its higher critical field [142]. The difference in quality factor of the niobium stripline in the copper and in the aluminum waveguide can be attributed to losses to the copper wall, as suggested by simulations. When increasing the temperature of the MSRs, two effects on the internal quality factors are expected. Approaching the critical temperature leads to a decrease of $Q_{\rm int}$, due to an increasing surface impedance. Considering a two fluid model [143], the following temperature dependence is found

$$\frac{1}{Q_{int}^{R_s}} = \frac{A}{T} exp\left(-\frac{\Delta}{k_B T}\right) + \frac{1}{Q_{\text{other}}}.$$
(6.1)

Here T is the temperature, Δ the superconducting gap at zero temperature, k_B the Boltzmann constant and A a constant. An additional Q_{other} accounts for other temperature independent losses. This model is expected to show good agreement until $T_c/2$. [144]

TLS saturate with increasing temperature, which leads to an increase in quality factor [129]

$$\frac{1}{Q_{\text{int}}^{\text{TLS}}} = k \tanh\left(\frac{hf_r(T)}{2k_BT}\right) + \frac{1}{Q'_{\text{other}}}.$$
(6.2)

Where k is the loss parameter and $hf_r(T)$ represents the energy of the TLS at the resonance frequency of the MSR for a given temperature. The resonance frequency barely changes with temperature (Fig. 6.7 b)), which allows us to fix the frequency of the TLS to the resonance frequency of the MSR in the low temperature limit. Q'_{other} is analogue to Eq. 6.1.

Figure . 6.6 b) shows the dependence of the internal quality factor of the aluminum MSRs on the base temperature of the dilution cryostat. The data is fitted to a combined model of TLS related losses (Eq. 6.2) and conductive losses (Eq. 6.1). Until about 200 mK the MSRs in the copper waveguide show a constant internal quality factor. This gives further evidence that dissipation due to TLS is not the dominant loss mechanism for the aluminum MSRs in the copper waveguide. In the aluminum waveguide an increase in $Q_{\rm int}$ with temperature until 200 mK is observed. Thus in this waveguide, TLS related losses most likely limit the quality factor of the MSR. Above 400 mK all MSRs show a similar decrease in $Q_{\rm int}$. This can be attributed to conductive losses, as the critical temperature, an

internal quality factor slightly above 1000 is measured. This is close to the results of finite element simulations, which predict an internal quality factor of about 500.



Figure 6.7: Temperature dependence of the Nb MSRs. \blacklozenge Nb MSR in Cu waveguide. \blacklozenge Nb MSR in Al waveguide. a) Internal quality factor of the niobium MSR at single photon powers. The data is fitted (lines) to the TLS model, Eq. 6.2. For the MSR in the aluminum waveguide, this model breaks down around 350 mK, when losses of the waveguide walls become dominating. Therefore data points above 350 mK are disregarded for the fits. b) Resonance frequency shift of the MSR at $\approx 10^6$ photons. The data is fitted to the model described by Eq. 6.3. In the copper waveguide good agreement is observed until around 800 mK (data point at 1 K is omitted for the fit). In the aluminum waveguide, above 350 mK the observed frequency change is dominated by the waveguide walls (as in a)).

6.4.2 Internal quality factors and frequency of Nb MSR with respect to temperature

Figure 6.7 a) shows the temperature dependence of the internal quality factor of the niobium MSRs. Niobium has a critical temperature of about 9.2 K [144], hence a breakdown of superconductivity isn't observed. Thus, the data is only fitted with the model describing TLS related losses (Eq. 6.2). The behavior of the MSR in the copper waveguide agrees well with predictions from theory throughout the whole measurement range. An increase of $Q_{\rm int}$ up to 1 K is observed. For the MSR in the aluminum waveguide a drop in the internal quality factor at 350 mK is measured. In this region the breakdown of superconductivity for the aluminum MSRs (Fig. 6.6 b)) is observed. This indicates that the breakdown of superconductivity in the waveguide walls is the limiting factor here. For higher temperatures, the internal quality factor remains approximately constant around 1×10^6 . Performing finite element simulations using the finite conductivity of the aluminum (Al5083 [145]) waveguide wall a $Q_{\rm int}$ of 1.16×10^6 is observed, which is consistent with our measurements. TLS also lead to a shift in the resonance frequency [129]

$$\Delta f_r(T) = f_r(0)\frac{k}{\pi} \times \left(\operatorname{Re}\Psi\left(\frac{1}{2} + \frac{1}{2\pi i}\frac{hf_r(T)}{k_BT}\right) - \log\left(\frac{1}{2\pi}\frac{hf_r(T)}{k_BT}\right)\right).$$
(6.3)

Here Ψ is the complex digamma function. Fig. 6.7 b) shows the frequency shift when increasing the temperature of the cryostat. In contrast to the effect on Q_{int} , off resonant TLS contribute to the frequency shift [129], which makes the resonance frequency independent of power. The only fit parameter is the combined loss parameter, k. For measurements of the Nb MSR in the aluminum waveguide, a drop in the frequency shift above 350 mK is observed. This again is due to the breakdown of superconductivity in the waveguide wall. Below 350 mK, the measurements are in good agreement with the model.

The values obtained for k by fitting the shift of the resonance frequency are about 10% to 30% lower, than fitting the change of the internal quality factor (Fig. 6.7 a)). This can be attributed to a non-uniform frequency distribution of TLS [129], which leads to a difference whether Q_{int} or Δf_r is considered. The intrinsic quality factor depends on losses to TLS near the resonance frequency, whereas the shift of the resonance frequency depends on a wider frequency spectrum of TLS.

An approximate low power, low temperature limit on Q_{int} is given by 1/k. Taking the k value found fitting the change of Q_{int} gives a 20% to 30% higher limit, than found in the measurements. This suggests that the majority of losses happen to TLS, but there is also a second loss mechanism. According to simulations, the internal quality factor of the MSR in the copper waveguide could be limited by the wall conductivity. In the aluminum waveguide it could be attributed to bulk dielectric loss from the high-resistivity silicon, as the loss tangent is not very well known [139].

The MSR in the waveguide represents a resonator in notch configuration [140]. For such a resonator, the S_{21} parameter, which refers to a transmission measurement, follows [140]:

$$S_{21}(f) = 1 - \frac{Q_l/|Q_c|e^{i\phi_0}}{1 + 2iQ_l \frac{f - f_r}{f_r}}$$
(6.4)

Here Q_l is the total quality factor, f_r is the resonance frequency and Q_c is the coupling quality factor. In here ϕ_0 accounts for an impedance mismatch in the transmission line before and after the resonator, which makes Q_c a complex number $(Q_c = |Q_c|e^{-i\phi_0})$. The real part of the coupling quality factor determines the decay rate of the resonator, in our case the emission to the waveguide. The physical quantity is the decay rate, κ , which is inversely proportional to the quality factor [146] and therefore the real part is found as: $1/Q_c^{Re} = \text{Re}(1/Q_c) = \cos \phi_0/Q_c$. Knowing Q_c^{Re} and Q_l , the internal quality factor can be obtained, as $1/Q_l = 1/Q_c^{Re} + 1/Q_{\text{int}}$ [146]. Plotting the imaginary versus the real part of S_{21} forms a circle in the complex plane (in case of a resonance within the frequency range).

Equation 6.4 represents an isolated resonator, not taking effects from the environment into account. Including the environment, which arises by including the whole measurement setup before and after the MSR, hence the equation is modified as .6.4 to [140]:

$$S_{21}(f) = \left(ae^{i\alpha}e^{-2\pi i f\tau}\right) \left(1 - \frac{Q_l/|Q_c|e^{i\phi_0}}{1 + 2iQ_l\frac{f-f_r}{f_r}}\right)$$
(6.5)

Here a and α are an additional attenuation and phase shift, independent of frequency. τ represents the phase delay of the microwave signal over the measurement setup, which has a linear dependence on frequency.

6.4.3 Tuning coupling of MSR to the waveguide

Fig. 6.8 illustrates the actually measured configurations. In the copper waveguide a) - e), one sample was measured each time, in the aluminum waveguide, three samples could be measured at once, labelled (f1)-(f3) in the following. The three samples in the aluminum waveguide were put along the propagation direction, all in the same configuration. As they MSRs had to have different resonance frequencies, one MSR had longer legs (f3), leading to a nominally lower resonance frequency of around 0.5 GHz. This MSR, as well as a second one (f2), having a resonance frequency of nominally 8 GHz, is backed with an empty silicon substrate. This reduces the resonance frequency, due to the higher effective dielectric constant.

The measurement results are plotted in Fig. 6.9, are in good agreement with the simulation data. The weak dependence on the number of photons agrees well with the expected power-independence of the coupling. The closer the MSR is to the wall, the lower the coupling quality factor a), b), inline with simulations. With the additional



Figure 6.8: Measured configurations of MSRs in the waveguide. The orange border refers to a waveguide made from copper, the gray border to one made from aluminum. In the copper waveguide, one sample was tested at each time, whereas in the aluminum waveguide three samples were tested at once, refereed to as (f1)-(f3). The dimensions of the different waveguides are stated. All dimension in mm.

empty substrate, the quality factors follow the predictions from simulations c), d). For the substrate further away from the MSR, d)- f), highest coupling quality factors are observed. The difference in the coupling between configurations d), e) and (f1), which should be similar (the only nominal difference is the waveguide width) to a slight displacement of the MSR as discussed before. The quality factors of (f2) and (f3) are higher, as they are already backed with an empty substrate. Thus the relative influence of the neighbouring substrate is reduced, leading to a higher Q_c .

6.4.4 Resonance frequency in dependence of photon number in the MSR

The measured resonance frequencies are shown in Fig. 6.10. The additional silicon substrate backing reduces the resonance frequencies of the 7.5 GHz(f3) and 8 GHz(f2) MSRs,



Figure 6.9: Coupling quality factor for different configurations (see Fig. 6.8). The lines depict the simulated data, the points are the measurements.



Figure 6.10: Resonance frequencies for the different configurations (see Fig. 6.8) in dependence of photon number. The lines depict the simulated data, the points are the measurements. a) All measured configurations. b) Configurations with nominally the same resonance frequency. The deviation of the simulated resonance frequency for configuration c) can be explained with the closer sapphire substrate, compared to the other setups. The sapphire leads to a higher effective ϵ_r and thus a lower resonance frequency.

plotted in (i). Except (f1), simulation results accurately predict the resonance frequencies. All the other setups have resonance frequencies in the same range (Fig. 6.10(ii)), which is predicted by simulations. There are several explanations for the 70 MHz deviation of the resonance frequency, which is not seen in the simulation data. One possibility is, a variance in the chip dimension. This would lead to a different effective dielectric constant and thus a lower resonance frequency. Other possibilities include a slight difference between the MSRs or its placement on the substrate.

There is no dependence of the resonance frequency on the number of circulating photons.

6.4.5 Shift of the resonance frequency of the Al MSR with increasing temperature



Figure 6.11: Shift of the resonance frequencies for increasing base temperature of the aluminum MSR. ▲ ■ Al MSR in copper waveguide. ▼ Al MSR in aluminum waveguide.

The shift of the resonance frequency of the three measured aluminum MSRs for increasing base temperature is plotted in Fig. 6.11. A decrease of the resonance frequency is seen above 500 mK. The shift is similar for all three measured samples. The drop in resonance frequency can be explained with an increasing surface inductance over temperature, which originates from an increasing effective penetration depth [58]. The increase of the penetration depth can be estimated with the Mattis Bardeen theory [147]. The results are similar to the one found in [59, 58], where thin aluminum film resonators were measured. There, the frequency shift shows good agreement with the Mattis-Bardeen theory.

6.4.6 Resonance frequency shift of the niobium MSR with increasing temperature - comparison between low and high input powers

Fig. 6.12 compares the shift of the resonance frequency for input powers at the single photon limit to input powers six magnitudes greater. The main difference is the higher noise in the single photon limit leading to increasing uncertainties. Overall, the low and high power measurements show the same temperature dependence. Thus both can be



Figure 6.12: Shift of the resonance frequency of the Nb MSR with increasing temperature. High power measurements were taken for the fit (Fig.6.6). $\Rightarrow \approx 10^6$ photons circulating in the resonator \Rightarrow single photon limit for the MSR in a copper waveguide. $\Rightarrow \approx 10^6$ photons circulating in the resonator \Rightarrow single photon limit for MSR in a aluminum waveguide.

taken to fit Δf_r with the same results. Given the lower uncertainties, the high power measurements allows to perform the fit.

In Fig. 6.12 the measurement results are plotted until 1.4 K. For the fit to the MSR in the copper waveguide, the data points above 0.8 K are omitted as the behavior above is not well described by the model anymore.

6.4.7 Internal quality factor of the niobium MSR for high excitation powers

Fig. 6.13 shows the internal quality factor of the niobium MSR over the whole measurement range. The best performing niobium MSR showed an internal quality factor of above eight million for high input powers. Due to attenuators in the measurement chain (Fig. 3.6 a)) higher input powers were not possible. Given the tendency, one would expect an even higher quality factor for higher input powers. The difference between the two MSRs in the aluminum waveguide is probably related to TLS losses. A slightly lower value for the combined loss parameter k of the better performing MSR (Tab. 6.2). The finite conductivity of the copper is probably the reason for the lower quality factor measured for the MSR in the copper waveguide.



Figure 6.13: Similar to Fig. 6.6 a) but showing the full measurement range of the niobium MSR. The internal quality factor reaches nearly ten million. ◆ Nb MSR in copper waveguide.
♦ Nb MSR in aluminum waveguide.

6.4.8 Fit results

The internal quality factor of the MSR changes with temperature. In case of the niobium MSR this is explained with the loss to two level systems (Eq.6.2). By fitting this model to the measurement data (Fig.6.7 a)). TLS also lead to a shift of the resonance frequency, predicted by Eq. 6.3. The measurement results including the fits are shown in Fig. 6.7 b).

In case of the aluminum MSR, an increasing surface resistance also leads to an additional effect on Q_{int} , next to the TLS. Thus a combined model of the surface impedance (Eq. 6.1) and TLS (Eq. 6.2) for the fit is used:

$$\frac{1}{Q_{\text{int}}^{\text{TLS} + \text{R}_{\text{s}}}} = k \tanh\left(\frac{hf_r(T)}{2k_BT}\right) + \frac{A}{T}exp\left(-\frac{\Delta}{k_BT}\right) + Q_{\text{other}}$$
(6.6)

To fit this model to the change of Q_{int} , the inverse of Eq. 6.6 was taken. The fit parameters of all performed fits are listed here. In Tab. 6.1 the fit results of Eq. 6.6 to the measurements of the aluminum MSR (Fig. 6.6 b)) are given.

In case of the aluminum MSR in the copper waveguide, 1/k can not be taken as a low energy low temperature limit for Q_{int} , as the MSR is limited by other losses. Only the MSR in the aluminum waveguide, being limited by TLS related losses, $1/k = 6(1) \times 10^5$, is in agreement with the measurements. In turn, it was not possible to extract a useful

Table 6.1: Fit results including fit errors for the measurement of the internal quality factor of the aluminum MSR, for stepwise increasing temperature. The results including the fits are shown in Fig. 6.6 b). The corresponding model is given in Eq. 6.6. Given that the the MSRs in the copper waveguide are not limited by TLS related losses, the fit values are not trustworthy. A similar conclusion can be drawn for the MSR in the aluminum waveguide, for Q_{other} . The MSR seems to be either limited by TLS or surface impedance losses, so Q_{other} is not seen in the measurement.

		A	$Q_{ m other}$	k
Al - cu	(🔻)	$5.6(7) \times 10^{-4}$	$4(1) \times 10^5$	$1.2(6) \times 10^{-6}$
Al - cu	(🔳)	$10.2(9) \times 10^{-4}$	$0.92(2) \times 10^5$	$4.46 \times 10^{-1} \pm 2.1$
Al - al	(🔺)	$6.5(7) \times 10^{-4}$	-	$1.6(2) \times 10^{-6}$

value for Q_{other} , as the MSR was either limited by TLS effects or increasing conductive losses. For the MSRs in the copper waveguide Q_{other} is in agreement with the measurements. The values obtained for A, which refers to the increasing surface impedance, are in the same range for all measurements.

Tab. 6.2 lists the fit results for the niobium MSR. The change of the internal quality factor with temperature (Fig. 6.7 a)) and the change of the resonance frequency with temperature (Fig. 6.7 b)) is fitted. For the niobium MSR the k can be either determined

Table 6.2: Fit results including fit errors for the measurement of the internal quality factor of the niobium MSR, for stepwise increasing temperature. The measurement results including the fits are shown in Fig. 4. The corresponding model for the change of Q_{int} is given in Eq.6.2 and for the change of the resonance frequency in Eq. 3.

		fit to Q	$Q_{\rm int}(T)$	fit to $\Delta f_r(T)$
		k	Q_{other}	k
Nb - cu	(•)	$10.0(2) \times 10^{-7}$	$2.53(4) \times 10^{6}$	9.1(5) $\times 10^{-7}$
Nb - al	(•)	$7.5(4) \times 10^{-7}$	$4.7(5) \times 10^{6}$	$5.6(3) \times 10^{-7}$
Nb - al	(•)	$7.9(7) \times 10^{-7}$	$7(2) \times 10^{6}$	$6.1(4) \times 10^{-7}$

by fitting the change of Q_{int} or the change of the resonance frequency. In both cases the value obtained are in the same range. Nevertheless, the value obtained fitting the resonance frequency is throughout 10% - 30% higher, than fitting Q_{int} . The reason lies in the frequency distribution of the TLS [129]. In addition, the Q_{int} limit given through k. Q_{other} gives an upper limit on the internal quality factor. The fit values are compatible with the measurements (Fig. 6.13).

6.5 Conclusion

The characterized MSRs show single photon intrinsic quality factor of up to one million at 20 mK. A strong dependence of the internal quality factors on the photon number and the temperature indicates losses to two level systems. The design presented is appealing for testing material of the MSR, the substrate it is patterned on and for validating fabrication processes. However, the observed quality factors are expected to increase when more complex designs are used, such as suspended structures [133] or by improving the surface quality through deep reactive ion etching [135]. Chapter 7

Publication 3: Bi-Stability in a Mesoscopic Josephson Junction Array Resonator

My contribution

I have taken the lead role in designing, characterization and the measurements described in this paper, and the writing of the manuscript. In collaboration with other authors, the measured data is analysed theoretically. We present an experimental investigation of stochastic switching of a bistable Josephson junctions array resonator with a resonance frequency in the GHz range. As the device is in the regime where the anharmonicity is on the order of the linewidth, the bistability appears for a pump strength of only a few photons. We measure the dynamics of the bistability by continuously observing the jumps between the two metastable states, which occur with a rate ranging from a few Hz down to a few mHz. The switching rate strongly depends on the pump strength, readout strength and the temperature, following Kramer's law. The interplay between nonlinearity and coupling, in this little explored regime, could provide a new resource for nondemolition measurements, single photon switches or even elements for autonomous quantum error correction.

7.1 Motivation

The non-linearity provided by atoms and Josephson junctions is a necessary ingredient to observe quantum mechanical effects in cavity quantum-electro-dynamics (QED) and circuit QED (cQED) systems. Strong nonlinearites, much larger than the linewidth of the transition, are required to realize qubits [30], implement quantum information protocols [148, 149] and realize textbook quantum optics experiments [150, 151]. Nonlinearities much smaller than the linewidth of the transition are typically exploited for parametric processes [45, 152, 36] like amplification or frequency conversion at the quantum level.

Besides quantum information applications, there has been a growing interest to exploit cavity QED for ultralow-power classical logic elements [153, 154, 155]. This interest has been sparked by the ever growing all optical communication networks. Remarkably, a single photon transistor [156], reminiscent of an electronic transistor, has been implemented for the optical domain. In this device a single photon can switch a large optical field. Realizing such devices has been a challenging endeavour as the required non-linearity is hard to realize, due to the weak interaction of optical light with atoms.

Much stronger light matter interactions can be achieved in the microwave regime using the cQED platform. In this context Josephson junction arrays (JJAs) have proven to be an ideal circuit element to build superconducting qubits with excellent coherence properties and unique tuning capabilities [157, 158, 159]. Similarly, JJAs have also been used to build quantum limited parametric amplifiers [160, 161, 36, 162]. Recently, the coherence properties of the self resonances of JJAs [163, 43], as well as their self-Kerr and cross-Kerr coefficients have been measured [51]. The measured Kerr coefficient showed good agreement with a model based on a second order expansion of the Josephson potential [164].

A regime of particular interest arises when the self-Kerr K_i and cross-Kerr K_{ij} nonlinear coefficients are on the order of the linewidth κ of the system. In this regime the system will show a pronounced bistability [165, 54] at the single to few photon level. Bistability is a phenomenon which is relevant in many fields, ranging from chemistry [166] and biology [167, 168] to Josephson junction physics [169, 170] and cQED [171]. Very recently, an optically levitated nanoparticle has been shown to exhibit a stochastic bistability [172] and Kramers turnover [173].

This chapter reports on the realization of a JJA resonator with multiple modes and strong self-Kerr and cross-Kerr coefficients in the regime $K_i, K_{ij} \approx \kappa$. Further by investigating the bistability of one mode of the JJA and characterize the dependence of the switching rate on the pump strength, readout strength and temperature. In addition, the numerical model, based on Kramer's' theory (described in the last section of chapter 2), is in good agreement with the experimental observations.

7.2 Device Description

The JJA consists of 10^3 cascaded Josephson junctions, with a small capacitance to ground C_0 , coupled to a 6 mm long microwave antenna and a shunt capacitance C_s , as shown in Fig. 7.1. The junctions are fabricated on a sapphire substrate using electron beam lithography and bridge-free double-angle evaporation [174]. An electron beam image of the junctions can be found in Fig. 7.1d. The junctions were designed to have a large ratio $E_J/E_C \approx 200$, in order to suppress coherent quantum phase slips (CQPS) [175, 43]. Here E_J is the Josephson junction energy and E_C is the charging energy. The parameters of the JJA were designed such that the fundamental resonance of the JJA combined with the shunt capacitance is around 1 GHz. The mode spacing for the first 10 modes is about 1.2 GHz and progressively becomes smaller for higher resonances [43, 51].



Figure 7.1: a) Photograph of one half of a rectangular copper waveguide with a 6 GHz cutoff. A JJA with a microwave antenna is fabricated on a piece of sapphire and placed in the center of the waveguide b) Schematic representation of an array of Josephson junctions inside a waveguide. C_0 is the capacitance of the islands to ground, C_S is the shunt capacitance for the array, C_J is the junction capacitance and C_g denotes the coupling capacitance of the antenna to the waveguide. L_J is the Josephson junction inductance. The input and output couplers to the waveguide are shown on the top left and top right of the schematic. (c) Optical image of the JJA coupled to a 6 mm long antenna and a shunt capacitance. The inset shows a zoom-in on the junction array. (d) Electron beam image (blue box in the inset) of ten of the 10^3 Josephson junctions.

The JJA is placed inside a copper waveguide [176] with a 6 GHz cutoff, as shown in Fig. 7.1a. Due to the capacitive coupling of the JJA to the waveguide, we can characterize the sample by performing microwave transmission measurements using a vector network analyzer (VNA). Due to the relative symmetry of the electric field of the waveguide and the antenna, the even modes of the JJA will couple poorly to the waveguide and not be visible in transmission measurements. The waveguide with the sample is mounted on the mixing chamber stage (10 mK) of a cryogen free dilution



Figure 7.2: Transmission measurements, measured with a power corresponding to about one photon circulating in the resonator. The solid line in a), (c), (d) is a fit using Eq. 7.1 to extract Q_{tot} and ω_R . a) Resonator response measurement for mode five b) Relaxation time T_1 on mode five with a pump strength of $\bar{n}_P = 115$ photons. The solid line is an exponential fit to the data with $T_1 = 3 \ \mu$ s. (c) Resonator response measurement for mode seven. (d) Resonator response measurement for mode nine.

refrigerator. The sample is enclosed in a double layer cryoperm shield inside a completely closed copper can. The stainless steel input lines are attenuated with 20 dB at 4 K and 30 dB at base temperature. They are filtered with a combination of a 12 GHz low pass and an Eccosorb filter. The output stage consists of a 12 GHz low pass filter, two 4-12 GHz isolators and a 4-8 GHz high electron mobility transistor amplifier. The effective measurement bandwidth for direct transmission measurements using a VNA is limited to about 4-9 GHz due to the cutoff of the waveguide and the combined bandwidth of other microwave components.

Within the accessible measurement bandwidth three resonances are characterized by fitting their transmission data to a notch type response function [177] given by eq.7.1. From these measurements the resonance frequencies ($\omega_i/2\pi$), the internal and the coupling quality factors is extracted shown Fig.7.2 and Table. 7.1.

$$S_{21} = 1 - \frac{\frac{1}{Q_{i,ext}} - 2i\frac{\delta f}{\omega_i/(2\pi)}}{\frac{1}{Q_{i,tot}} + 2i\frac{\omega - \omega_i}{\omega_i}}.$$
(7.1)

The parameter δf takes into account an impedance mismatch. The extracted parameters are summarized in Table 7.1.

Mode	$\omega_i/2\pi$	$Q_{\rm tot}$	κ_i	K_i	K_{i7}	
	(GHz)		(kHz)	(kHz)	(kHz)	
5	4.816(1)	26000	181	66	187	
7	7.1058(2)	950	7500	133	-	
9	9.278(1)	3375	2750	218	343	

Table 7.1: Parameters for the three array modes that can be directly measured with the VNA. f_r and Q_{tot} were extracted from data. The Kerr and cross-Kerr coefficients are calculated from a fit to the dispersion relation Fig. 7.3d.

Fig. 7.2b shows a measurement of the decay time T_1 of mode five for a pump strength of about $\bar{n}_P = 115$ photons. To excite the resonator the pump was detuned by about -2.54 MHz from the bare frequency $\omega_P/2\pi = 4.8156$ GHz of mode five. Similar to a two-tone measurement we use mode seven as a readout, with a readout strength of $\bar{n}_R = 0.5$ photons. To perform the decay time measurement we excite mode five for a few μ s with a pulse before performing a readout on the pump mode again using a pulse of a few μs . By varying the delay between the two pulses we measure how the excitation decays over time. From the measurements we find $T_1 \approx 3\mu s$.

To observe the resonances of the modes outside the measurement bandwidth, we exploit the cross-Kerr interaction, which is induced by the junction non-linearity. The Hamiltonian for the JJA, up to second order, is given by

$$H/\hbar = \sum_{i=1}^{N} (\omega_i a_i^{\dagger} a_i + \frac{K_i}{2} a_i^{\dagger} a_i a_i^{\dagger} a_i) + \sum_{\substack{i,j=1\\i\neq j}}^{N} K_{i,j} a_i^{\dagger} a_i a_j^{\dagger} a_j.$$
(7.2)

The Hamiltonian consists of a self-Kerr term K_i which leads to a photon number $n_i = a_i^{\dagger} a_i$ dependent frequency shift of mode *i* and a cross-Kerr interaction K_{ij} , which leads to a frequency shift of mode *i* depending on the photon number in all other modes *j*.
7.3 Two-tone spectroscopy

Since the other resonance frequencies of the JJA's are out of the HEMT amplification bandwidth or below the cutoff frequency of the waveguide, we utilize a two-tone measurement technique to indirectly measure them. We use mode seven as read-out and apply a second pump tone. We sweep the frequency of this pump tone from 960 MHz to 20 GHz. Such a two-tone spectroscopy measurement can be seen as a pump probe experiment. The VNA is used to monitor one mode while a signal generator is used to excite a second one. We then observe the shift in the read-out mode due to populating other resonances through the pump tone. From this we can observe e.g. the fundamental mode of the array at 963 MHz and other higher resonant modes up to 20 GHz which is the limit of our signal generator. Two tone measurements for our array are shown in Fig 7.3.

Figure 7.3d, shows the dispersion relation of the array. The frequencies outside the HEMT bandwidth are extracted by two-tone spectroscopy measurements as shown in Fig.7.3a, b, c. The mode spacing for the first 10 modes is about 1.2 GHz. For higher resonant modes the spacing between modes becomes smaller. The \bullet in Fig 7.3d are obtained by diagonalizing the capacitance matrix which includes the shunt capacitance (C_s) , junction capacitance (C_J) , the ground capacitance (C_0) and the Josephson inductance (L_J) for the entire array structure. From the fit the obtained JJA parameters are $C_0 = 0.152$ fF, $C_J = 34$ fF, $L_J = 1.25$ nH, $C_S = 18$ fF with a confidence range of about 20 %. These parameters match well to the expected design values and room temperature resistance measurements of the junctions. With these parameters, the selfkerr K_i and the cross-kerr $K_{i,j}$ can be calculated using a procedure similar to Ref. [51]. The results are summarized in Table 7.1 of for modes five, seven and nine. Using these coefficients the photon number is calibrated by measuring the resonance frequency shift as a function of the applied power as shown in Fig. 7.5. The attenuation extracted from these Kerr measurements matches within 4% to the attenuation in the cryostat, determined by independent transmission measurements. Most of the even modes can not be observed from two tone measurements as the electric field distribution of these modes has a symmetry that does not couple to the waveguide. For higher mode numbers this symmetry is somewhat broken due to inhomogeneities of the junctions and we can again excite these modes. It should also be noted that a discrepancy between the model



Figure 7.3: Two-tone measurements and dispersion curve. a),b) and (c) show the transmitted amplitude of the readout mode with frequency $\omega_R/2\pi = 7.105$ GHz as a function of the pump tone frequency ω_P^D and detuning Δ_R . The black arrows mark the frequencies when the pump tone matches a mode of the resonator, ω_P^D , leading to frequency shift in the read out tone. (d) Measured resonant mode frequencies of the JJA from the two-tone spectroscopy. • represents the calculated dispersion relation without including corrections due to the cross coupling between segments of the array, and • represents the measured resonant mode frequencies up to 20 GHz.

and the measurements for modes 9 and 10. This is most likely due to the capacitive cross coupling between the parallel segments of the chain of Josephson junctions 7.1 c. This effect can be accounted for by introducing additional capacitance's with a minimal impact on the values of the self-Kerr and cross-Kerr coefficients for the lower modes.

7.3.1 Two-tone spectroscopy on bi-stable mode

Figure 7.4 show the results of a two-tone spectroscopy where the frequency of a pump tone around mode five is swept while weakly probing mode seven with the VNA. When the pump tone is resonant with mode five, the resonance frequency on mode seven shifts due to the cross-kerr interaction. The measured frequency of mode five, using two-tone spectroscopy, matches the direct VNA measurement.



Figure 7.4: Two-tone measurement. a) Shift Δ_R of the resonance frequency of the readout mode $\omega_R/2\pi = 7.105$ GHz upon application of a pump tone. The pump tone is detuned from the resonance $\omega_P/2\pi = 4.8156$ GHz of the 5th mode of the JJA by a detuning Δ_P and has a pump strength of $\bar{n}_P = 115$ photons. Due to the nonlinearity of the JJA, the resonance frequency of the readout mode is shifted when the pump tone matches mode five. b) Zoom in on the bistable region of mode five. For a detuning of about $\Delta_P = -2.58(4)$ MHz two different resonance frequencies of the readout mode can be observed. The shifted and unshifted resonances correspond to 115 and 1 circulating photons in mode five, respectively.

Upon closer inspection of the two-tone scan in Fig.7.4a one can observe a bistable region (see Fig.7.4b) for a detuning of $\Delta_P = -2.58$ MHz from the bare resonance frequency of mode five. Around this frequency, two different cross-Kerr shifts of mode seven can be observed: A shift of $\Delta_R = -7.32$ MHz corresponding to a photon number of about 115 photons in mode five and a shift of 133 kHz corresponding to about one photon. In the two-tone scan the residence times exceeding ten seconds in either the high or low photon number states has been observed.

7.4 Kerr coefficients and photon number calibration

7.4.1 Self-Kerr measurements

For the self-Kerr measurements only one mode i of the chain is excited. The frequency shift Δ_P with respect to the input power is measured using direct transmission measurements. Each data-set is fitted with a notch type response function to extract the frequency ω_i and $Q_{i,\text{tot}}$. From the shift relative to the bare resonance frequency the self-Kerr coefficient K_i is extracted.



Figure 7.5: Self-Kerr and cross-Kerr measurements. The error on each data point is about point size. a) Dependence of the self-Kerr frequency shift on the input power of mode five. The red line is a third order polynomial fit - see text. b) Dependence of the self-Kerr frequency shift on the input power of mode seven. The red line is a linear fit - see text. (c) Cross-Kerr frequency shift of mode seven when applying input power to mode five. The red line is calculated using the theoretical prediction for the linear self-Kerr term - see text. The read-out mode seven is driven with about $\bar{n}_R \approx 0.5$ photons. (d) Cross-Kerr frequency shift of mode five when applying input power to mode seven. The red line is a polynomial fit to the data. The read-out mode five is driven with about $\bar{n}_R \approx 1$ photons.

Fig. 7.5a shows the self-Kerr measurements on mode five. For low input power P the resonance frequency Δ_5 changes linearly with power. For high input power, higher order

Kerr terms (K'_i, K''_i) start to play a role. To take this into account the data is fit with the following dependence. With this the Kerr coefficients are extracted in units of Hz/W.

$$\Delta_i(P) = K_i P + \frac{K_i'}{2} P^2 + \frac{K_i''}{3} P^3$$
(7.3)

Fig. 7.5b shows the self-Kerr measurements on mode seven. Here the resonance frequency Δ_7 changes linearly with the power due to the higher linewidth (κ_7) of this mode.

7.4.2 Cross-Kerr measurements K_{75} utilizing two-tone spectroscopy

The cross-Kerr coefficients K_{ij} are determined by utilizing two-tone spectroscopy measurements. One mode i of the array is used as the readout with a power of about one photon and the other mode j is excited with varying power. From the shift Δ_R of the resonance frequency of the readout mode $\omega_R/2\pi$ upon application of a pump tone, the cross-Kerr shift K_{ij} is extracted. Figure 7.6 a) shows a typical two-tone spectroscopy measurements to extract the cross-Kerr coefficient K_{75} by pumping mode five and observing the frequency shift on mode seven. Mode seven is driven with a constant readout strength of $\bar{n}_R = 0.5$ photons. To extract the maximal frequency shift Δ_R , each pump frequency of the data is fit to a notch type response function and extract the resonance frequency(solid black line). The maximal shift of the readout resonator for a given pump power results in one datapoint in Fig. 7.5 c). Figure 7.6 b)-c) shows the two-tone spectroscopy for different pump powers. From the measurement in Fig. 7.6 one can also clearly observe that the mode becomes bistable as the power is increased. Figure 7.5 c) shows the result of all two tone measurement to determine the cross-Kerr shift K_{75} when driving mode five and using mode seven as the readout. For low input power the resonant frequency Δ_7 changes linearly with the power but then rapidly higher order terms come into play. In this case, even a third order polynomial fit does not agree with the measured K_{75} . Thus the cross-Kerr coefficient has been characterized by fitting the slope at low powers in Fig. 7.5c. By using the dispersion relation fit in Fig. 7.3d and the resulting diagonalized capacitance matrix, the ratio between K_5 and K_{75} is extracted. This ratio, together with K_5 determined from the linear part of the fit function Fig. 7.5a allows us to compute K_{75} . A linear fit with the computed slope K_{75} , shown by the solid red line in Fig.7.5c, shows good agreement with the measurements at low powers.



Figure 7.6: Cross-Kerr measurements K_{75} . **a**, **b**, **c**, **d** Two-tone spectroscopy measurements for different pump strength. Shift Δ_R of the resonance frequency of the readout mode $\omega_R/2\pi =$ 7.105 GHz with $n_R = 0.5$ photons upon application of a pump tone to mode five. The pump frequency is detuned by Δ_P from the resonance of mode five $\omega_p = 4.1856$ GHz. **a**),**b**), (**c**) and (**d**) Corresponds to a pump power of ≈ 0.01 nW, ≈ 0.7 nW, ≈ 1 nW and ≈ 2 nW respectively; the solid black line corresponds to fit where the resonance frequency of the readout mode for each pump frequency is extracted. From the fits the maximal frequency shift of the read-out mode is extracted, for a given pump power in mode five.

Fig. 7.5d shows the cross-Kerr measurements K_{57} on mode five using mode seven as the readout. Here the third order polynomial fit agrees well with the data. From the ratio of the first order self and cross-Kerr coefficients extracted from fitting the data in Fig. 7.5b and Fig. 7.5d and compare it to the Kerr coefficients extracted from fitting the dispersion relation, the Kerr coefficients are in excellent agreement.

Furthermore, by converting the Kerr coefficients extracted from Fig. 7.5a-d from Hz/W to Hz/photon the required conversion factor matches within 4% to the attenuation in the cryostat, determined by independent transmission measurements.

For 0.1 nW input power it is estimated to be about 10 photons in mode five. By applying this same conversion factor to the whole polynomial Eq. 7.3 and then use the observed

frequency shift for a given input power to calculate the circulating photon number.

7.4.3 Power dependence of the line-width

Fig. 7.7 shows the linewidth of mode five and seven with respect to the input power. The line-widths are extracted from direct transmission measurements. Each measurement is fit with a notch type response function to extract the resonance frequency f_r and Q_{tot} for a given drive power. The increase of the line-width κ_i as a function of the circulating power is expected from the self-Kerr effect. For a circulating power of less than one photon in mode five we find $\kappa_5(\bar{n}_P \to 0) = 181$ kHz, and for mode seven $\kappa_7(\bar{n}_P \to 0) = 7.5$ MHz.



Figure 7.7: a) Line-width κ_5 vs. input power on mode five. For the lowest drive power we still have a good enough signal to noise ratio to fit the data. We find $\kappa_5(\bar{n}_P \to 0) = 181$ kHz. The black solid line corresponds to a fit of a $\sqrt{\text{Power}}$ dependency. b) Line-width κ_7 vs. input power on mode seven. On the lowest drive power we still have a good enough signal to noise ratio to fit the data with a $\kappa_7(\bar{n}_P \to 0) = 7.5$ MHz.

7.5 Continuous time measurements

To precisely characterize the residence time, a readout scheme similar to the dispersive state detection of a superconducting qubit in a circuit QED architecture [22] is implemented. Here mode seven is monitored continuously on resonance with a readout power \bar{n}_R of about half a photon such that the mode does not shift or broadening. When mode five is pumped with \bar{n}_P photons, a change in the transmitted readout signal is observed corresponding to jumps between the high and the low amplitude states. Each data point is averaged on 500 measurements lasting 19.9 μ s to get a good signal to noise ratio. In addition, the analog to digital converter needs another 5 ms to transfer the data. Thus, it takes about 15 ms to acquire one data point. The measurement routine using the SDR14 acquisition board is shown in figure 7.8. Figure 7.9a shows a typical



Figure 7.8: Measurement routine using SP device's SDR 14 800 Msamples/s acquisition board. Total measurement time of 19.9 μ s times the number of averages (500 averages in our case). In addition, the ADC converter needs 5-6 ms of data transfer time to transfer the data.



Figure 7.9: a) Transitions from the low amplitude state to the high amplitude state for $\bar{n}_P =$ 9 photons at a detuning of $\Delta_P \approx \Delta_P^{\text{Max}} = -0.54(1)$ MHz from mode five. The readout mode seven was driven with $\bar{n}_R = 0.5$ photons on resonance. The trace displayed here is a 15 s segment out of a total recorded time of 20000 s. b) Histogram of the amplitude distribution $\rho(A)$ for the data displayed in a. (c) Histogram of $\rho(A)$ for a detuning of $\Delta_P = -0.59(1)$ MHz.(d) Histogram of the amplitude distribution $\rho(A)$ for a detuning of $\Delta_P = -0.49(1)$ MHz. (e). Dependence of the normalized state population in the high and low photon state on Δ_P relative to the bistable point. The solid line is a fit to a sigmoid function.

time trace for a measurement time of 15 s and $\bar{n}_P=9$ photons in mode five. One can clearly observe two distinct amplitudes in transmission corresponding to two distinct photon numbers in mode five. The switching rate Γ in this case is defined as the inverse of the average time between two transitions from the low to the high power state. The transient time between these two states is much faster than the data acquisition rate. Similar bistable behaviour was also observed in co-planar waveguides but in a different parameter regime, where the non-linearity was much smaller than the mode linewidth [171]. For larger non-linearities, fewer photons are required for the bifurcation offering promising opportunities for the development of microwave components at the single photon level. In addition, the dynamics of the system is accurately modeled using Kramers' theory.

A histogram of the data shows a well separated bi-modal distribution for the two amplitude states. Furthermore, the residence times are extracted in the high amplitude state, the resulting histogram shows an exponential behaviour typical for a Poissonian statisics (see Fig.7.13). When the pump frequency is swept Δ_P across the bistable point the relative height of the peaks changes in the amplitude distribution $\rho(A)$, Figure 7.9be. There is a Δ_P^{Max} for a given pump strength where the heights of the peaks are equal, as shown for example in Fig.7.9b, and the switching rate is maximal.

7.5.1 Switching rate (Γ) dependence on the pump strength

In this section, the switching rate measurements for a total of ten different pump powers ranging from 2.6 to 115 photons in mode five has been discussed. Figure 7.10b shows the plot for maximally achieved switching rate Γ_{Max} and the corresponding pump detuning Δ_P^{Max} versus photon number. The lowest power where the switching can be observed for is $\bar{n}_P = 2.6$ photons at a detuning of -169 kHz from the low power resonance. This detuning matches well to the prediction [73] of $\Delta_P = (\frac{\sqrt{3}}{2}) \kappa_5 = 160$ kHz with $\kappa_5 = 181$ kHz.

Typically, stochastic switching in a bistable system is described by Kramers theory [178] (please refer to the theory chapter 2.7 for detail explanation). There, the switching rate is determined by the potential landscape and the fluctuations. For a symmetric potential it is given by $\Gamma_{\text{Max}} = \Gamma_0 \exp(-E_b/k_bT)$, where E_b is the barrier height between the two stable solutions and the prefactor Γ_0 which depends on the relative strength of the dissipation and the potential [178]. Here, the activation of the switching between the two stable solutions likely originates from the dispersive shift of the resonator frequency due to photon number fluctuations in the readout mode and thermal fluctuations of the photon number in all modes.

In our case, the potential landscape is created by the interplay between the pump, the self-Kerr effect and the damping of the mode. An intuitive choice for this potential [165, 54] is provided by integrating the equation for the photon number in the steady state

of a damped Kerr oscillator [179]. From this model the scaling of E_b [180] and Γ_0 for the maximum switching rate is extracted as a function of the pump photon number \bar{n}_P . Further using this scaling in a fit function (see Fig. 7.10b) with two free parameters to match our data. In addition $\Gamma_{\rm res}$ is taken into account the finite switching rate for high pump powers. This finite rate could be limited by phase slips on the junctions of the JJA, which is estimated to be in the range of a few mHz.

Additionally, we observe a change of the switching rate with respect to Δ_P following a lorentzian curve. The point of maximum switching rate Δ_P^{Max} also corresponds to a symmetric amplitude distribution. In Fig.7.10a one can see the change of the switching rate with respect to Δ_P for three different pump powers. The width of this lorentzian is about 72 kHz and the center shifts with increasing photon number in the pump. From our model the shift of Δ_P^{Max} with \bar{n}_P agrees well with the experiment as shown in Fig. 7.10b. The deviation at high photon numbers can be explained by higher order Kerr effects which are not taken into account in the model.

In order to compare the experimental results with the theoretical approach above, we first note that the parameter η_P discussed in the theory chapter 2.7 is not directly known but has to be inferred from the circulating photon number \bar{n}_P . Let us consider the situation in 7.10b, where the maximum switching rate and the associated detuning are plotted as a function of \bar{n}_P at the point of the maximum switch. In order to compare to the above model, by varying η_P in the bistable region $[\eta_+, \eta_-]$ (with K_{PP} and κ_i chosen as the values in the experiment) the detuning $\tilde{\Delta}_P^{\text{Max}}$ is first numerically located, for a given value of \bar{n}_P satisfying the symmetric potential condition namely $U(\bar{n}_{+}) = U(\bar{n}_{-}) = U(\bar{n}_{P})$. After adding a constant shift of $K_{PP}/2 + K_{PR}\bar{n}_{R}$ to our numerically determined $\tilde{\Delta}_{P}^{\text{Max}}$, as shown by the solid red line in Fig. 7.10b, the experimental result are in good agreement with theory. From Eq.2.66, the barrier height E_b , and frequencies ω_L and ω_0 for this symmetric point as a function of \bar{n}_P are extracted. In order to fit the switching rate to Kramers equation, a form for the pre-factor Γ_0 is added to our equation. It is found that in general the form $\Gamma_0 \propto \omega_L \omega_0$ valid for the over-damped regime fits best to the experimental data. Fitting the measured rates Γ_{max} to the functional form $A\omega_L\omega_0 \exp(-\beta_{fit}E_b) + \Gamma_{res}$ on a log-log scale using least squares procedure, the obtained fit parameters $\Gamma_{res} = (0.006 \pm 0.002)$ Hz, $A = (54.0 \pm 15.0)$ Hz and the effective temperature $\beta_{fit} = (2.4 \pm 0.2)$. The errors quoted here are the standard deviation on the estimated best fit parameters. The fitted curve was depicted by the



Figure 7.10: a) Dependence of switching rate Γ on Δ_P for three different pump strengths: •- $\bar{n}_P = 2.6$ photons, •- $\bar{n}_P = 4$ photons, •- $\bar{n}_P = 6$ photons. The solid lines are lorentzian fits to the data. b) Extracted Γ_{Max} and corresponding Δ_P^{Max} as a function of \bar{n}_P . The black and red lines are from a fit to our theoretical model (see text). (c) Γ measured for constant $\bar{n}_P = 6$ photons while varying the readout strength: •- $\bar{n}_R = 0.5$ photons, •- $\bar{n}_R = 1.5$ photons, •- $\bar{n}_R = 2.5$ photons. (d) Γ measured for different cryostat base temperatures with $\bar{n}_P = 6$. In a), b) and (d) $\bar{n}_R = 0.5$.

black line in 7.10b. We reiterate that in the fitting procedure the parameters E_b , ω_L and ω_0 were calculated numerically from the potential Eq. 2.66.

7.5.2 Switching rate (Γ) dependence on the readout strength

To better identify the origin of the switching we also varied the power in the readout tone. The measured results are plotted in Fig.7.10c for constant $\bar{n}_P = 6$ photons. We can observe two effects for an increasing photon number in the readout mode: I) due to the cross-Kerr effect the bistable point moves to lower frequencies. II) in contrast to lowering the switching rate with the pump photon number, the readout photon number increases the switching rate. For $\bar{n}_R = 2.5$ photons we see an increase in the switching rate by about a factor of three. This can be understood as a form of measurement induced dephasing. As the photon number in the readout resonator fluctuates, the position of the bistable point moves in frequency due to the cross-Kerr effect. This moves the pump in and out of the bistable region into the regime where either the low or the high power state are more likely, as the cross-Kerr frequency is on the order of the linewidth. This photon number fluctuation happens at a rate $\kappa_7 = 7.5$ MHz and is thus much faster than our acquisition time. For $\bar{n}_P > 2.5$ photons, the switching becomes much faster and we cannot observe it any more due to the limited measurement bandwidth and signal to noise. Remarkably, changing \bar{n}_R by only one photon for a constant Δ_P we can switch the state of mode five from the low to the high photon number occupation.

7.5.3 Switching rate (Γ) dependence on the temperature

To study the influence of thermal noise on the switching rate, we increased the cryostat base plate temperature from 10 mK to 50 mK ($\bar{n}_P = 6$ photons, $\bar{n}_R = 0.5$ photons). Increasing the temperature increases the average thermal population as well as the fluctuations of the photon number in all of the modes of the JJA, most notably for the lower frequency modes. As a consequence, depicted in Fig. 7.10d, we observe a shift in the magnitude and location of the maximum switching rate, similar to Fig. 7.10c. This is again due to the cross-Kerr interactions of mode five with all other modes. The observed shift can be explained, by an increase in the JJA temperature up to about 100-130 mK depending on the initial temperature. We can get an upper bound for the JJA temperature at the base temperature of the fridge, if we assume that the linewidth broadening we observe for mode five is due to the thermal population in the other modes of the JJA. This broadening can be explained by a minimal temperature of the JJA of ≈ 50 mK. This is consistent with other cQED experiments [181, 182], where the devices are well above the fridge base temperature.

7.5.4 Additional observation

In addition to the bi-stability, at an intermediate power of about $\bar{n}_P = 4 - 9$ metastable behaviour has been observed on the mode 5 of the JJAR. Figure 7.11 shows one such plot for a pump strength of about $\bar{n}_P = 4$ photons, while the readout mode seven was further driven with $\bar{n}_R = 0.5$ photons. However, the metastable behaviour is not investigated in detail.



Figure 7.11: Continuous time measurement: a) Transitions from low amplitude state to the intermediate metastable states and to the high amplitude state for $\bar{n}_P = 4$ photons on mode five. The readout mode seven was driven with $\bar{n}_R = 0.5$ photons.b) Histogram of the amplitude distribution for the data displayed in a).

7.5.5 Width of bistable region for varying photon number



Figure 7.12: Width W_{Γ} of the bistable region for different pump strengths \bar{n}_p .

Figure 7.12 shows the width of the bistable region obtained from the switching rate measurements for the ten different pump strengths shown in Fig. 4b in the main text.

For very low pump strengths up to 10 photons the width of the bistable region is approximately constant ≈ 72 kHz. For photon numbers greater than 10 photons the bistable region becomes wider.

7.5.6 Residence time and state population inversion

Figure 7.13a, b show the exponential dependence of the residence time for a pump strength of $\bar{n}_P = 9$ photons and $\bar{n}_P = 6$ photons at a detuning of $\Delta_P = \Delta_P^{Max}$ MHz from mode five. This indicates that the transitions are random and follow a Poissonian statistics. From the exponential fit to the data, a mean residence time is extracted in the high state $\langle T_{UP} \rangle$.



Figure 7.13: a), b). Probability of being either in the low-photon state or in the high-photon state in a bistable region. •, • represents the measured data, — , — are from fits using Eq. 2.67, and — ·, — · are fits to a sigmoid function. The pump strength is $\bar{n}_p = 9$ for a) and $\bar{n}_p = 6$ for b). The inset in a), b) shows the Histogram of the residence time $T_{\rm UP}$ with pump strength of $\bar{n}_p = 9$, $\bar{n}_p = 6$ and for a detuning of $\Delta_P = \Delta_P^{Max}$ MHz from mode five. The red line is an exponential fit to the data giving $\langle T_{\rm UP} \rangle = 14.4$ s, $\langle T_{\rm UP} \rangle = 6.5$ s.

The inset plots in Fig.7.13a,b show the probability for being either in the low or high photon state by scanning Δ_P across the bistable region for the different pump strengths $\bar{n}_P = 9$ photons and $\bar{n}_P = 6$ photons. It shows the state population inversion between the low photon and the high photon state, following a sigmoid behaviour $f(x) = 1/(1 + e^{-x})$.

The statistics of the results for different \bar{n}_p , for both the residence time and the probability distributions, are consistent with bistable systems well described by the Kramers model [71].

7.6 Conclusion

A stochastic bistability in a 10^3 Josephson junction array which appears at a pump strength of only a few photons is measured. This switching at low power is achieved by engineering the Kerr interaction strength to be comparable to the linewidth. An exponential decrease of the maximal switching rate for increasing pump strength as expected from Kramers theory is observed. For an increase in readout strength or temperature, the switching rate increases, likely due to photon induced dephasing through cross-Kerr interactions.

Chapter 8

Qubit readout using a Josephson junction array resonator

In this chapter I describe a Josephson junction array resonator (JJAR.2.0) engineered for a quantum non-demolition measurement (QND) on a qubit. The intention of the engineered device is to perform a high efficiency qubit readout without any dephasing on the qubit. In the end of this chapter I show some preliminary results, characterizing the Josephson junction array resonator and the qubit.

8.1 Motivation

Quantum information using superconducting circuits requires qubits with long coherence times combined with a high-fidelity readout. A common strategy to readout a qubit consists in coupling it dispersively to a resonator, so that the qubit states $|0\rangle$ and $|1\rangle$ shift the resonance frequency differently. The frequency change can be detected by measuring the phase or amplitude of a microwave pulse being either reflected or transmitted through the resonator. However, this readout scheme faces two difficulties which prevent from measuring the qubit state in a single readout pulse: the first one is that the readout has to be completed in a time much shorter than the time T_1 in which the qubit relaxes from $|1\rangle$ to $|0\rangle$ and the second with a power low enough to avoid spurious qubit transitions [183]. A possible route of solving faster readout scheme is by using a sample-and-hold detector consisting of a bi-stable hysteric device [33, 47, 48]. In a sample-and-hold detector system, the bi-stable resonator is a Josephson bifurcation amplifier. The $|g\rangle$ and $|e\rangle$ state of the qubit is mapped on to the two stable states respectively [47, 184]. When the JBA device is driven by a microwave signal at a chosen frequency and power, this non-linear resonator bifurcate between two stable states with different intra-cavity field amplitude and reflected phases [48, 47]. The initial design of JBA's are used to readout quantroniums and flux-quits, obtaining fidelity up to 90% with QND character. The major drawbacks of the JBA are: to have the resonator bifurcate at a certain power will cause unwanted qubit state transitions during readout [48, 47] and the resonator is filled with ($\bar{n} > 100's$) photons to achieve bi-stability which leads to excess back-action on the qubit [73].

In another technique, the qubit state is mapped to the oscillator state of a parametricphase locked oscillator(PPLO) [46]. A PPLO is a resonant circuit in which one of the reactances is periodically modulated. It can detect, amplify, and store binary digital signal in the form of two distinct phases of self-oscillation [185]. The qubit readout scheme using a PPLO enables a fast and latching-type readout. Once the qubit state is latched to the oscillator state of the PPLO, as long as the pump of the PPLO is turned-on the projected state of the qubit is protected irrespective to subsequent qubit transitions. This scheme requires only a small number of readout photons in the resonator to which the qubit is coupled, unlike the sample-and-hold detector system. [46].

In this chapter, our approach is to engineer an array of Josephson junctions and utilize the lowest two resonant modes of the JJAR.2.0 for qubit readout. The fundamental mode of JJAR.2.0 is coupled to the qubit and used as a standard dispersive read-out. Due to symmetry coupling between the qubit and the JJAR.2.0, the qubit couples only to the odd modes of the JJAR.2.0 and decoupled from the even modes of the JJAR.2.0. Due to the cross-Kerr interaction, the second mode of the device is coupled to the fundamental mode. The second mode of the JJAR.2.0 is pumped with a few hundred photons to exhibit bi-stability. The qubit state can be determined using a readout tone on mode two of the JJAR.2.0 via the pump tone on mode one. The main goal of this device is to engineer the qubit-JJAR coupling, and to maximize measurement speed and achieve single-shot QND readout of the qubit.

8.1.1 Design constraints and working principle

The working principle of the device is described in this section. In order to achieve an efficient qubit readout resonator, listed below are the few constraints that are taken into account for engineering our device.

• Odd and even modes of the Josephson junction array must couple to a rectangular waveguide with linewidth in the order of a few $\kappa \approx MHz$. The qubit is coupled dispersively to mode one of the JJAR.2.0 while mode two of the JJAR.2.0 is decoupled from the qubit. For dispersive readout of qubit the anharmonicity of the fundamental mode has to be smaller than few MHz.

• The second mode of the array should exhibit bi-stability at a few photons. For this reason the second mode of the device is engineered to have an anharmonicity (α) to be around a few MHz's.

• The rectangular waveguide acts as a high pass filter. Above the cutoff frequency most of the energy will pass through the waveguide. Below the cut-off frequency, the electric field decay's exponentially and the energy is attenuated by the waveguide [56]. In our design, I utilize this extra attenuation of the waveguide by having the qubit transition frequency below the cut-off frequency of the waveguide [186], protecting the qubit from the spontaneous emission known as Purcell effect [187, 74]. The qubit is perpendicular to the electric field of the waveguide, which decouples the qubit from the waveguide. This is useful to protect the qubit from dephasing induced by residual thermal photons and hence improves the coherence time of qubit. Residual thermal photons are suspected to arise from noise impinging on the readout resonator [186].

After carefully engineering the device by full-filling all the above mentioned constraints. The working principle of the device is described in two steps which are given as follows (see illustration 8.1):

1) Since the first mode of the array is dispersively coupled to the transmon qubit, the resonance frequency of mode one can change based on the state of the qubit $|g\rangle$ (red dot) and $|e\rangle$ (blue dot) [22]. The second mode of the array is decoupled from the qubit and probed with a few hundred photons to show bi-stability (\bar{n}_2). Due to the cross-Kerr interaction the frequency of mode two changes when the first mode is excited with a few



Figure 8.1: Sketch illustrating the working principle of the qubit readout using JJAR.2.0 in a rectangular waveguide. The qubit is dispersively coupled to mode 1 of the array and is decoupled from the second mode of the array. The second mode of the array is pumped with $(\bar{n}_2 > 100's)$ to be bi-stable. For QND-readout, mode two of the array is used to readout the state of the qubit via the pump tone resonant with mode one. The pump on mode one shifts the resonant frequency of mode two by the amount of photons (χ_{12}) pumped into mode one (\bar{n}_1) . Due to high photon number in mode two, it latches onto one of the two bistable states depending on the state of the qubit. When the qubit is in the ground state $|g\rangle$, low signal amplitude is measured on mode two, and if the qubit is in excited state $|e\rangle$ high signal amplitude on mode two is measured.

photons. The amount of frequency shift is equivalent to the cross-Kerr coefficient times the number of photons pumped on mode one (\bar{n}_1) .

2) The pump $(\bar{n}_1 > 0)$ is used to enable qubit readout via mode two of the array. The pump is tuned in resonance with the mode one (ω_P) when the qubit is in the ground state (green arrow in the illustration). The power on mode one is calibrated to drive mode one with a few photons to minimize unwanted back-action on the qubit, but sufficient to shift mode two by a few MHz. The readout tone is chosen at a particular resonant frequency on mode two ω_R (orange arrow in the illustration). When the qubit is in the ground state, mode one is affected by the pump and mode two is shift lower in frequency by a few MHz (dotted line position) and the readout signal is transmitted with low amplitude as shown in the illustration 8.1. Due to the hysteresis of the bi-stability, mode two will stay locked at this frequency due to the high circulating photon number. If the qubit is instead in the e-state, the pump will not excite mode one and mode two will remain at a high frequency (solid lines in the illustration). The readout signal will now be transmitted with higher signal amplitude with no circulating power ($\bar{n}_2 = 0$) in mode two.

8.2 Theory of a transmon qubit coupled to a JJAR.2.0

In this section I discuss the theory of a transmon qubit and the Hamiltonian of a JJA coupled to a transmon qubit. The Hamiltonian of the system is given as follows

$$\hat{H} = \hat{H}_q + \hat{H}_{array} + \hat{H}_{int} \tag{8.1}$$

The first and second term in the Hamiltonian 8.1 is the effective Hamiltonian of a transmon qubit and a JJAR. The last term in the Hamiltonian is the interaction between them.

8.2.1 Theory of a transmon qubit

The transmon consists of two superconducting islands separated by a Josephson junction [50]. The superconducting islands are designed such that the capacitive energy of the circuit is lowered. In a circuit representation, depicted in figure 8.2, a large shunt capacitance C_{sq} is added in parallel to the Josephson junction. As a result the total capacitance is given by $C_{\sum} = C_{Jq} + C_{sq}$. Where C_{Jq} and C_{sq} are the junction capacitance and the shunt capacitance of a transmon qubit. The enhanced capacitance stabilises charge fluctuations [50].



Figure 8.2: a) Circuit representation of a transmon qubit. The cross in the box symbolises the non-linear inductance including the capacitance. The capacitance C_{sq} is parallel to the Josephson junction in order to lower the capacitive energy E_C of the transmon qubit. b) The potential energy of a transmon is periodic and has a cosine shape. Hence the quantized levels are not equidistant in energy.

$$H_q = \frac{Q^2}{2C_{\sum}} - E_J \cos\phi. \tag{8.2}$$

Where E_J is the Josephson energy, $C_{\sum} = C_{Jq} + C_{Sq}$ is the sum of capacitance's. The transmon qubit is an anharmonic oscillator and acts as a qubit for a weak drive strength, with the transition frequency $(\hbar \omega_q)$ between ground and excited state. The properties of the transmon are discussed in detail in [50]. For large E_J/E_C the effective phase across the junction is small. Thus we can approximate the cosine by a taylor expansion up to fourth order. The taylor expanded eq. 8.2 is given as

$$H_q \approx \frac{Q^2}{2C_{\sum}} - E_J + \frac{E_J}{2}\phi^2 - \frac{E_J}{24}\phi^4.$$
 (8.3)

The non-linear behaviour is still included in the fourth order term. The first three terms approximate the cosine as a parabola describing a harmonic oscillator with a constant energy shift E_j . The constant energy shift is neglected, because energy differences are measured. The phase difference ϕ is connected to the flux variable Φ via $\phi = \frac{2\pi\Phi}{\Phi_0}$, hence the charge and flux variable in equation 8.3 is replaced by the canonical conjugated operators which are given as follows.

$$\hat{\Phi} \approx i(b - b^{\dagger}),$$

$$\hat{Q} \approx (b + b^{\dagger})$$
(8.4)

Where b^{\dagger} and b are the creation and annihilation operator, respectively. They satisfy $[b, b^{\dagger}] = 1$. These operators are chosen such that they obey the canonical commutation relation $[\hat{\Phi}_n, \hat{Q}_n] = i$. By substituting the canonical conjugate operators 8.4 in eq. 8.3. The Hamiltonian eq. 8.3 is rewritten as follows

$$\hat{H}_q = \hbar \omega_0 \hat{b}^{\dagger} \hat{b} - \frac{e^2}{24C_{\sum}} (\hat{b} - \hat{b}^{\dagger})^4.$$
(8.5)

 ω_0 is the transition frequency of the qubit between the ground and excited state. Expanding $(\hat{b} - \hat{b}^{\dagger})^4$ and apply the first order perturbation theory and using the commutation

relation $[\hat{b}, \hat{b}^{\dagger}] = 1$. The resulting Hamiltonian is

$$\hat{H}_{q} = \hbar\omega_{0}\hat{b}^{\dagger}\hat{b} - \frac{E_{C}}{2}(\hat{b}^{\dagger}\hat{b})^{2} - E_{C}\hat{b}^{\dagger}\hat{b}.$$
(8.6)

The transmon is driven weakly such that only between lowest two energy levels are taken into account with transition frequency ω_0 . The number operator $\hat{b}^{\dagger}\hat{b}$ is replace by the Pauli operator σ_z . The final Hamiltonian of the qubit is given by

$$\hat{H}_q = \frac{\hbar\omega_q}{2}\sigma_z \tag{8.7}$$

where $\hbar\omega_q = \hbar\omega_0 - E_c$. Since the transmon qubit is an anharmonic oscillator, the anharmonicity $\alpha = E_{12} - E_{01}$ (where E_{ij} is the energy difference between energy state j and i) as shown in sketch 8.3, is measured by exciting the qubit at high power [188].



Figure 8.3: Energy transitions of the transmon qubit. On the left a dipole transition between the ground $|g\rangle$ and the first excited state of the qubit $|e\rangle$ is shown. On the right a two photon transition between the ground $|g\rangle$ and the second excited state of the qubit $|f\rangle$ is shown. The energy difference between this energy state is $\alpha/2$

This additional transition is a two photon transition. Two photons at a slightly lower frequency $(\alpha/2)$ excite the second level of the qubit with low probability. Thus very high power is used increasing the probability of two photon transition to happen. The frequency difference between the single and two photon transition is used to obtain information on the anharmonicity.

In the transmon regime [50] the anharmonicity α equals the charging energy $\alpha = -E_C$. Since the qubit transition frequency ω_q and the charging energy is known, the Josephson energy E_J is calculated by

$$\hbar\omega_q = \sqrt{8E_C E_J} - E_C \tag{8.8}$$

8.2.2 Hamiltonian of JJAR.2.0

The electrical circuit of JJAR.2.0 is shown in figure 8.4a. The Lagrangian for the circuit shown in 8.7a is given by



Figure 8.4: JJAR.2.0. a) Electrical circuit representation of the JJAR.2.0, C'_0 is integrated at the center of the array without breaking the symmetry. b) show the eigen-phase for the fundamental mode (*black dots*) and second mode of the array (*red dots*) obtained from the numerical simulations. The discrepancy due to influence of additional center ground capacitance C'_0 is clearly visible on the 9th JJ. c) Dispersion relation for the circuit. The lowest mode of the array is influenced by the shunt capacitance(C_S), C'_0 influence the even modes of the array, having the largest influence on the second mode of the array.

$$\mathcal{L} = \frac{C_S}{2} \dot{\Phi}_0^2 + \frac{C_0'}{2} \dot{\Phi}_{N/2}^2 + \sum_{x=1}^{N-1} \left(\frac{C_0}{2} \dot{\Phi}_x^2 \right) + \sum_{x=0}^{N-1} \frac{C_j}{2} (\dot{\Phi}_{x+1} - \dot{\Phi}_x)^2 - \sum_{x=0}^{N-1} E_J \left(1 - \cos\left(\frac{2\pi}{\Phi_0} (\Phi_{x+1} - \Phi_x)\right) \right)$$
(8.9)

In order to transform to the Hamiltonian the conjugate momenta (charges) are derived with respect to the node fluxes defined on the islands Φ_x

$$Q_{N/2} = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}_{N/2}} = \dot{\Phi}_{N/2} C_0 + (\dot{\Phi}_{N/2} - \dot{\Phi}_{N/2-1}) C_j + \dot{\Phi}_{N/2} C_0'$$
(8.10)

$$Q_x = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}_x} = \dot{\Phi}_x C_0 + (\dot{\Phi}_x - \dot{\Phi}_{x-1})C_j - (\dot{\Phi}_{x+1} - \dot{\Phi}_x)C_j$$
(8.11)

Rewriting the charges in a matrix representation

$$\vec{Q} = \hat{C}\vec{\dot{\Phi}} \tag{8.12}$$

With the derivative of the flux vector with respect to time $(\vec{\Phi}^T = (\dot{\Phi}_0, \dot{\Phi}_1, ..., \dot{\Phi}_N))$ and the capacitance matrix

$$\hat{C} = \begin{pmatrix}
C_0 + C_j + C_s & -C_j & 0 & \cdots & \\
\vdots & 0 & \ddots & \ddots & \ddots & \ddots & \\
0 & -C_j & C_0 + C_j + C'_0 & -C_j & 0 & \cdots \\
\vdots & 0 & \ddots & \ddots & \ddots & \ddots & \\
0 & 0 & \vdots & -C_j & -C_j + C_0
\end{pmatrix}$$
(8.13)

combined with the inverse of the inductance matrix

$$\hat{L}^{-1} = \begin{pmatrix} \frac{2}{L_J} & \frac{-1}{L_J} & 0 & \cdots & \\ \vdots & 0 & \ddots & \ddots & \ddots & \ddots \\ 0 & \frac{-1}{L_J} & \frac{2}{L_J} & \frac{-1}{L_J} & 0 & \cdots \\ \vdots & 0 & \ddots & \ddots & \ddots & \ddots & \ddots \end{pmatrix}$$
(8.14)

One can rewrite the Lagrangian of equation 8.9 as

$$\mathcal{L} = \frac{1}{2}\vec{\Phi}^{T}\hat{C}\vec{\Phi} - \frac{1}{2}\vec{\Phi}^{T}\hat{L}^{-1}\vec{\Phi}$$
(8.15)

By performing the Legendre transformation using the momentum vector

$$\vec{Q}^T = (Q_0, Q_1, Q_2, ..., Q_N)$$
(8.16)

to obtain the Hamiltonian of the Josephson junction chain in the linear limit

$$H = \vec{Q}^T \vec{\Phi} - \mathcal{L} = \vec{Q}^T \hat{C}^{-1} \vec{Q} - \frac{1}{2} \vec{Q}^T \hat{C}^{-1} \vec{Q} + \frac{1}{2} \vec{\Phi}^T \hat{L}^{-1} \vec{\Phi}$$
$$= \frac{1}{2} \vec{Q}^T \hat{C}^{-1} \vec{Q} + \frac{1}{2} \vec{\Phi}^T \hat{L}^{-1} \vec{\Phi}$$
(8.17)

Since the Hamiltonian is quadratic, it can be diagonalized and represented in the form [51, 52]

$$H = \frac{1}{2} \sum_{K=0}^{N-1} \hbar \omega_K a_K^{\dagger} a_K \tag{8.18}$$

By introducing the non-linearity of the Josephson junction as a perturbation the total Hamiltonian of the JJA is given as follows (please refer to the theory chapter 2.5.2 for detailed derivation).

$$\hat{H}_{array} = \sum_{j} \hbar \omega'_{j} \hat{a}^{\dagger}_{j} \hat{a}_{j} - \sum_{j} \frac{\hbar}{2} K_{jj} (\hat{a}^{\dagger}_{j} \hat{a}_{j})^{2} - \sum_{\substack{j,k\\(j \neq k)}} \frac{\hbar}{2} K_{jk} \hat{a}^{\dagger}_{j} \hat{a}_{j} \hat{a}^{\dagger}_{k} \hat{a}_{k}$$
(8.19)

Where K_{jj} and K_{jk} are the self and cross-Kerr coefficients.

8.2.3 Hamiltonian of total system



Figure 8.5: Circuit of a transmon qubit capacitively coupled to a Josephson junction array resonator which is loaded with an extra ground capacitance C'_0 and shunt capacitance C_{S1} and C_{S2} . C_g is the coupling capacitance between the qubit and the array. As the transmon couples only symmetrically to the array and to the asymmetric odd modes (the coupling capacitance C_g on left and right are the same).

The electrical circuit of our system is shown in figure 8.5. The interaction between a transmon qubit and a single resonant cavity mode gives rise to the well known Jaynes-Cummings Hamiltonian. The effective Hamiltonian of a JJAR coupled to a qubit is given by equation 8.20 [50].

$$\hat{H} = \frac{\hbar\omega_q}{2}\hat{\sigma}_z + \sum_j \hbar\omega'_j \hat{a}^{\dagger}_j \hat{a}_j - \sum_j \frac{\hbar}{2} K_{jj} (\hat{a}^{\dagger}_j \hat{a}_j)^2 - \sum_{\substack{j,k\\(j \neq k)}} \frac{\hbar}{2} K_{jk} \hat{a}^{\dagger}_j \hat{a}_j \hat{a}^{\dagger}_k \hat{a}_k + \hbar g (\sigma_+ + \sigma_-) (a_1 + a^{\dagger}_1)$$
(8.20)

The first term describes the qubit as a two level system or qubit. This **approximation** is valid, only when the qubit is driven on a single transition between two of its states only. The two states are usually referred to as $|g\rangle$ and $|e\rangle$ for the ground and excited state, respectively.

The non-trivial dynamics of the system arise due to a last term term in the Hamiltonian, accounting for the coupling of the atomic dipole moment of the qubit and the cavity electric field (first mode of JJAR.2.0). The coupling strength g determines the strength of the interaction. By ignoring fast oscillating terms the interaction Hamiltonian is given as follows [50].

$$\hat{H} = \frac{\hbar\omega_q}{2}\hat{\sigma}_z + \sum_j \hbar\omega'_j \hat{a}^{\dagger}_j \hat{a}_j - \sum_j \frac{\hbar}{2} K_{jj} (\hat{a}^{\dagger}_j \hat{a}_j)^2 - \sum_{\substack{j,k\\(j \neq k)}} \frac{\hbar}{2} K_{jk} \hat{a}^{\dagger}_j \hat{a}_j \hat{a}^{\dagger}_k \hat{a}_k + \hbar g (\hat{a}_1 \hat{\sigma}^{\dagger} + \hat{a}_1^{\dagger} \hat{\sigma}^{-})$$
(8.21)

In the dispersive regime $(\Delta \gg g)$ [22], since the transmon couples symmetrically to the array and to the asymmetric odd modes. I consider the coupling between the qubit and the fundamental mode of the array and the cross-kerr interaction between the second mode and first mode of the array, the effective Hamiltonian 8.21 is modified as follows:

$$\hat{H} = \hbar \left(\omega_1 + \frac{K_{11}}{2} (\hat{a}_1^{\dagger} \hat{a}_1) + \frac{K_{12}}{2} (\hat{a}_2^{\dagger} \hat{a}_2) + \chi \sigma_z \right) (\hat{a}_1^{\dagger} \hat{a}_1) + \hbar \omega_2 \hat{a}_2^{\dagger} \hat{a}_2 + \frac{\hbar}{2} K_{22} (\hat{a}_2^{\dagger} \hat{a}_2)^2 + \frac{\hbar}{2} \omega_q \sigma_z$$
(8.22)

Where χ is the dispersive shift on mode one depending on the state of the qubit.

8.3 Device description

The engineered device is fabricated on a TOPSIL silicon substrate with a thickness of 530 nm, using electron beam lithography and bridge-free double-angle evaporation for Josephson junction array resonator. The fabricated JJAR is shown in figure 8.6 e [82] and the cross-junction for the transmon qubit in figure 8.6 d. The device consists in total of 18 Josephson junctions in series, integrated with asymmetric shunt capacitance pads as shown in figure 8.6 c. The JJAR.2.0 is divided into two half's by introducing an extra ground capacitance C'_0 at the center [35]. The additional ground capacitance influence's the even modes of the array resonator, reducing the resonant frequencies of



Figure 8.6: Device description. a) Photograph of one half of a rectangular waveguide with a 6 GHz cutoff. A JJA and a transmon qubit is fabricated on a piece of silicon substrate and placed in the corner of the waveguide. b) Circuit representation of a transmon qubit coupled to a rectangular waveguide via a Josephson junction array. C_{S1}, C_{S2} is the coupling capacitance between the JJA and the rectangular waveguide, C_g is the coupling capacitance between the qubit and JJA. c) Optical image of the JJAR.2.0 and a transmon qubit. d) Zoom-in SEM image of a cross-junction design for transmon qubit. e) Zoom-in SEM image of a 9 Josephson junctions array.

the even modes. The parameters of the JJAR were designed such that the fundamental frequency of the JJAR.2.0 combined with the shunt capacitance is around 6 GHz. The

lowest two modes of the JJAR.2.0 are below 10 GHz and the spacing between the modes is about 2 GHz. The plasma frequency of the JJAR.2.0 is around $\omega_{plasma} \approx 30$ GHz.

The junctions in the array resonator were designed to have a large ratio of $E_J/E_C \approx 313$. The sample is mounted in a rectangular waveguide with a cutoff frequency of 6 GHz. The reason to have the sample off-center of the waveguide axes is to have the linewidth of the resonant modes (κ_k) approximately equal to the anharmonicity (K_k). In this regime the system show bi-stability at very few photons. Figure 8.6 b show the circuit representation of the qubit coupled to the waveguide via a JJAR.2.0. A transmon qubit is coupled capacitively (C_g) to the JJA resonator, and the JJA resonator is coupled capacitively via the shunt capacitance pads (C_{S1}, C_{S2}) to the waveguide. Please refer to Appendix B for the detail fabrication procedure of the sample.

8.3.1 Finite element simulations



Figure 8.7: Finite element simulations: a) User-defined mesh on the device. The two red lines show where the two arrays of 9 Josephson junctions each are located. b) Electric field of the fundamental mode of JJAR.2.0 coupled to a transmon qubit. The electric field has its maximum intensity on the left and right pads of JJAR.2.0 with a minimum node at the center of structure, hence coupling strongly to the dipole of the qubit. c) Electric field of the second mode of JJAR.2.0, the gradient of field is perpendicular to the qubit, resulting in the qubit being decoupled from the second resonant mode of the array.

Fine tuning the parameters of a JJAR.2.0 geometry requires finite element simulations with HFSS to find the correct parameter range for the resonance frequency of the modes $\omega_r(k)$, total capacitance of the structure (C_{Σ}) and the inductance (L_a) of the circuit. Since the array consists in total of 18 junctions I assign the characteristic impedance for each junction to a lumped circuit element in HFSS (red color lines in figure 8.7 a)). A fine mesh on the lumped elements is required to have accurate results. Figure 8.7 a) show the user-defined mesh on a JJ array resonator. The centre capacitance pad is the extra ground capacitance which reduce the resonant frequencies of the even modes. Due to asymmetric shunt capacitance (C_{S1}, C_{S2}) pads both the odd and even modes of the device couples efficiently to the waveguide.

Figure 8.7 b) show the magnitude and vector electric field (from finite element simulations) for the fundamental mode of the device. The electric field has its maximum intensity (red color) on the right and left pad and a node in the center of the device representing a $\lambda/2$ resonator. The resulting gradient along the qubit pads enables the coupling between the qubit and the fundamental mode. The voltage difference on the qubit clearly show that the fundamental mode of the array can be used to excite the qubit.

Figure 8.7 c) show the magnitude and vector electric field for the second mode of the device. The maximum intensity of the electric field is at the left, the right, and the middle pad of the resonator and the field is oscillating symmetrically with respect to the center of the device. Since the resulting gradient of the field is perpendicular to the qubit, the second mode of array is decoupled from the qubit.

8.4 Experimental results

The waveguide with the sample is mounted on the mixing chamber stage (10 mK) of a cryogen free dilution refrigerator. The sample is enclosed in a double layer cryoperm shield inside a completely closed copper can. The stainless steel input lines are attenuated with 20 dB at 4 K and 30 dB at base temperature. They are filtered with a combination of a 12 GHz low pass and an Eccosorb filter. The output stage consists of a 12 GHz low pass filter, two 4-12 GHz isolators and a 4-8 GHz HEMT amplifier. The effective measurement bandwidth for direct transmission measurements using a VNA is limited to about 4-9 GHz due to the cutoff of the waveguide and the combined bandwidth of other microwave components.



Figure 8.8: Transmission measurements, measured with a power corresponding to about one photon circulating in the resonator. The solid line in the plot is a fit using Eq. 8.23 to extract Q_{tot} and ω_R . a) Resonator response measurement for mode one. b) Resonator response measurement for mode two.

8.4.1 Transmission measurements

Since the sample is capacitively coupled (C_{S1}, C_{S2}) to a rectangular waveguide as shown in figure 8.6 a), it can be characterized by performing transmission measurements on the waveguide using a VNA. The modified design of JJAR.2.0 couples both the odd and even modes of the JJA to the waveguide. From the transmission S_{21} of the waveguide, the resonant modes which are in the bandwidth of the experiment can be measured easily. Two resonant modes out of 18 resonances of the array are shown in figure 8.8. The measured raw data is further fitted with the following equation to extract the quality factors of the resonances [43]

$$S_{21} = 1 - \frac{\frac{1}{Q_{ext}} - 2i\frac{\delta f}{f_R}}{\frac{1}{Q_{tot}} + 2i\frac{f - f_R}{f_R}}.$$
(8.23)

Where Q_{tot} , Q_{ext} is the total and the coupling quality factor and δf takes into account the impedance mismatch between the waveguide and the resonator. The fundamental frequency of the JJAR is around 5.8 GHz with quality factors ($Q_{int} = 30000, Q_{ext} =$ 14000) and the second mode of the array is around 7.5 GHz with quality factors ($Q_{int} =$ 50000, $Q_{ext} = 12000$).

8.4.2 Two-tone spectroscopy



Figure 8.9: Two-tone measurements. a) show the transmitted amplitude of the probe tone with frequency $\omega_R/2\pi = 5.799$ GHz as a function of the pump tone frequency ω_P^D and detuning Δ_R . When the pump tone frequency comes in resonance with the probe, the probe tone shifts in frequency. The shift in probe frequency at $\omega_P^D/2\pi = 5.799$ GHz corresponds to self-kerr of the mode one. The frequency shift on the probe tone at $\omega_P^D/2\pi = 7.599$ GHz corresponds to the cross-kerr between mode one and on the mode two. b) As the device is intentionally engineered to have the third resonant mode at around 30 GHz, there are no other shifts visible up to bandwidth of the experimental setup which is 20 GHz. This is to cross check and calibrate the device.

From the direct transmission measurements on the waveguide, both the first mode and the second mode of the JJA are measured using a drive power of a single photon in the resonant modes. To characterize the device, I utilize a two-tone spectroscopy measurement to find other resonant modes of the array, to cross check there are no other resonances up to bandwidth of the experimental setup which is 20 GHz. Figure 8.9 a), b) show the results of a two-tone spectroscopy measurement, where the frequency is swept from 0 to 20 GHz with a constant pump tone while weakly probing the fundamental mode $\omega_R/2\pi = 5.799$ GHz of the array with a VNA. By comparing the measured frequencies of the entire JJA with the re-normalized mode frequencies calculated by diagonalizing the capacitance matrix (Appendix A), the JJA parameters can be extracted $C_0 = 0.152$ fF, $C_J = 30$ fF, $L_J = 0.755$ nH, $C_S = 85$ fF and $C'_0 = 255$ fF.

8.4.3 Kerr measurements

The Kerr effect manifests itself as a frequency shift that depends linearly on the number of photons in a resonant mode [52, 51, 44]. By varying the input power on a low signal

$$n = \frac{2Q_{tot}^2}{\hbar\omega_k^2 Q_{ext}} P_{in} \tag{8.24}$$

Here ω_k is the resonance frequency of a particular mode, Q_{tot} is the total quality factor, Q_{ext} is the coupling quality factor and P_{in} is the input signal power. Further by fitting the measured data with a third order polynomial, the anharmonicity of each mode can be extracted.

8.4.3.1 Self- Kerr measurement

For the self-Kerr measurements only one mode i of the chain is excited. Further measuring the frequency shift Δ_P with respect to the input power using direct transmission measurements. Each data-set is fitted with a notch type response function to extract the frequency ω_i and $Q_{i,tot}$. From the shift relative to the bare resonance frequency the self-Kerr coefficient K_i is extracted.



Figure 8.10: Self-Kerr measurements. a) Dependence of the self-Kerr frequency shift on the photon number of mode one. The red line is a third order polynomial fit. b) Dependence of the self-Kerr frequency shift on the photon number of mode two. The red line is a third order polynomial fit.

Figure a, b show the self-Kerr measurements on mode one and mode two of the JJAR.2.0. For low input power P, the Δ_i changes linearly with power. For high input power, higher order non-linearity terms (K'_1, K''_1) start to play a role. To take this into account the data is fitted with the following dependence.

$$\Delta_i(\bar{n}) = K_1 \bar{n} + \frac{K_1'}{2} \bar{n}^2 + \frac{K_1''}{3} \bar{n}^3 \tag{8.25}$$

Here K'_1, K''_1 corresponds to the higher order non-linear terms. Thus extracting the Kerr coefficients in units of Hz/Photon.

8.4.3.2 Cross-kerr measurements

For determining the cross-Kerr coefficients K_{ij} , I utilize two-tone spectroscopy measurements [52, 43]. Mode 1 of the array is used as the readout with a power of about one photon and mode 2 is excited with varying power and vice-versa. From the shift Δ_R of the resonance frequency of the readout mode $\omega_R/2\pi$ upon application of a pump tone, the cross-Kerr shift K_{ij} can be extracted.

Figure 8.11 a show a typical two-tone spectroscopy measurements to extract the cross-Kerr coefficient K_{12} by pumping mode two and observing the frequency shift on mode one. Mode one is driven at a constant readout strength of $\bar{n}_1 < 1$ photons. To extract the maximal frequency shift Δ_1 , each pump frequency is fitted to a notch type response function 8.23 and the resonance frequency(solid black line) is extracted. The maximal shift of the readout resonator for a given pump power results in one data point in Figure 8.11 b), c), d). From the measurement in figure 8.11 c one can also clearly observe that the mode becomes bistable at a few hundred's of photons.

Figure 8.12 a), b) show the result of all two tone measurement to determine the cross-Kerr shift K_{12} when driving mode two and using mode one as the readout and K_{21} when driving mode one and using mode two as the readout [51]. For low input power, Δ_{21} changes linearly with the power but then rapidly higher order terms come into play. Hence the data is fitted to a third order polynomial. From the fit, the cross-Kerr coefficient can be extracted as K_{12} , $K_{21} = 49$ KHz/Photon. By inserting the measured values of self-Kerr coefficients of the mode one and the mode two of the JJAR.2.0 in the theoretical formula $(1/\sqrt{2}\sqrt{K_{11}K_{22}})$, I find an agreement within 20 %.



Figure 8.11: Cross-Kerr measurements K_{12} . a), b), c), d) Two-tone spectroscopy measurements for different pump strength. Shift Δ_R of the resonance frequency of the readout mode $\omega_R/2\pi = 5.799$ GHz with $n_R = 1$ photons upon application of a pump tone to mode two of JJAR.2.0. The pump frequency is detuned by Δ_P from the resonance of the second mode $\omega_P/2\pi = 7.599$ GHz. a), b), c), d) Corresponds to different number of photons ≈ 8 , ≈ 35 , ≈ 230 and ≈ 400 photons in the pump mode.

The measured Kerr coefficients of the device for mode one and mode two of device is summarized in the table below and compared to the analytical calculations obtained by diagonalizing the capacitance matrix (refer to appendix A).

The measured anharmonicities (α_2) of the JJAR.2.0 resonant modes is not equal to the linewidth (κ_2) of the resonant mode, thus bi-stability on second mode appears at a few hundred's of photons. This is mainly due to influence of the linear geometric inductance coming from the Josephson junctions and the connecting wires of array shown with green color in figure 8.13 b).



Figure 8.12: Summary of Cross-Kerr measurement. a) Cross kerr measurement, where mode one of the arrays is pumped and mode two of the array is used to readout the frequency shift. b) Mode two of the arrays is pumped and mode one of the array is used to readout the frequency shift. The red line is a third order polynomial fit.

Mode	$\omega_i/2\pi$	Q_{int}	Q_{ext}	Q_{tot}	κ_i	K_i	K_{i2}	$K_i(Theory)$	$K_{i2}(Theory)$
	(GHz)				(kHz)	(kHz)	(kHz)	(MHZ)	(MHz)
1	5.799(3)	30000	14000	10680	543	59.2	49	6.247	6.755
2	7.599(3)	50000	12000	9364	811.5	120.7	-	7.35	-

Table 8.1: Parameters for the two modes of JJAR.2.0 that can be directly measured with the VNA. f_r and Q_{tot} were extracted from data. The Kerr and cross-Kerr coefficients are calculated from a fit to the polynomial fit as shown in the Fig. 8.12, 8.10.

8.4.4 Influence of the geometric inductance

The measured Kerr non-linearities of the JJAR.2.0 are low compared to the numerical calculations given in table 8.1. This is due to not taking into account the influence of linear inductance present in the circuit 8.13 a). The geometric inductance influences the participation ratio of JJAR.2.0, resulting in lower non-linearities. The geometric inductance of a thin film wire is calculated with the following equation [189].

$$L_P = 2 * 10^{-3} * l * \left[\ln \left(\frac{2.0 * l}{w + t} \right) + 0.5 + 0.2235 \left(\frac{w + t}{l} \right) \right] \mu H$$
(8.26)

Where l, w, t is the length, width and thickness in cm. In addition to Josephson inductance $L_J = 0.755$ nH, the JJAR is mostly influenced by the geometric inductance of thin films arising from thin wires of the Josephson junctions (green lines shown in figure 8.13 b)) which is around $L_P = 0.18$ nH (calculated from equation 8.26). The additional geometric linear inductance is in series to the Josephson inductance. Hence



Figure 8.13: a) Electric circuit representing the influence of geometric linear inductance (L_P) arising from Josephson junction array resonator coupled to a transmon qubit. b) Zoom-in SEM image, total of 18 Josephson junctions array divided into two half's with additional ground capacitance (C'_0) at the center. The green lines on the Josephson junctions contribute the extra linear distributed geometric inductance (L_P) .

the circuit diagram 8.5 is modified with an additional linear inductance in series to each Josephson junction as shown in figure 8.13 a). By adding the linear inductance (L_P) to the numerical calculations, I find an agreement of the measured anharmonicity within 28%.

In order to reduce the geometric inductance, it is important to reduce the Josephson junction length and increase the width of the Josephson junctions. By doing this the Josephson inductance will be kept constant and reduce the geometric inductance of the thin films. The plan for the future device is to reduce the effect of linear inductance arising from thin film of the Josephson junction.

8.4.5 Qubit measurements

The transmon qubit in our design is perpendicular to electric field of the waveguide. Hence the qubit is decoupled from the waveguide. Since the transmon is designed symmetrically to the JJAR.2.0, thus it only coupled to the asymmetric odd modes of the array and decoupled from the even modes. The fundamental mode of the JJAR.2.0 is dispersively coupled to the qubit and is used to drive the qubit. The optical image of the transmon qubit coupled to the JJAR.2.0 is shown in figure 8.6 c).
The transmon qubit in this case is 1.2 mm long with an inductance of $L_q = 19$ nH and a total capacitance of a few tens of fF 's. The asymmetric pads of the JJAR.2.0 shown in figure 8.6 c) allow the qubit to couple strongly to the fundamental mode of the array. Hence the information about the state of a qubit can only be readout using the fundamental mode of the JJAR.2.0. By utilizing two-tone spectroscopy measurement it is possible to determine the transition frequency of the qubit.



Figure 8.14: a) Two-tone spectroscopy measurement: The transmitted amplitude of the readout mode one of the JJAR.2.0 with frequency $\omega_R/2\pi = 5.799$ GHz as a function of the signal generator frequency and detuning Δ_R . b) High power excitation measurement. Exciting the qubit at high power reveals a transition to the second excited state at a frequency of 3.647(4) GHz.

Figure 8.14 a) show the two tone measurement on the qubit. Since the qubit is coupled only to mode one of the JJAR.2.0. I utilize a two-tone spectroscopy measurement to measure the transition frequency of the qubit. The frequency of the pump tone is swept around the qubit frequency, while weakly probing mode one with the VNA. When the pump tone is resonant with the qubit transition frequency, the resonance frequency of mode one shift as shown in figure 8.14 a).

From the measurement the transition frequency of the qubit is $\omega_q/2\pi = 3.768(4)$ GHz. The initial qubit design is intended to have a transition frequency around 5 GHz. The frequency of the transmon qubit is off by about 1 GHz compared to the finite element simulations. This is due to a wrong estimate of the superconducting energy gap Δ (estimated $\Delta = 210$ eV) of our Aluminium(Al) material. It turned out that the energy gap of our superconducting Al material used for fabricating the device is about 165 eV, hence effecting the Josephson inductance of transmon qubit. The anharmonicity of the qubit is measured by exciting the qubit at high power. Due to the high power an additional transition appears as shown in figure 8.14 b), exciting the qubit's next higher state. The additional transition is a two photon transition. The difference between the single and the two photon transition is used to gain information on the anharmonicity of the qubit. The characterized parameters of the qubit are given in the table below.

Parameter	Value	Error	HFSS	HFSS
			(with superconducting	(with superconducting
			$gap(\Delta) = 210 \text{ eV})$	$gap(\Delta) = 165 \text{ eV})$
Qubit frequency (GHz)	3.7681	2e-5	5.08	4.1351
Anharmonicity (MHz)	241.4	0.2	261.5	248.43
Chi (MHz)	0.8	0.08	1.8	1.17
L_J (nH)	19.77	0.02	13	19
g(MHz)	78.17	0.05	94.5	125.7
E_J (GHz)	8.268	0.007	12.5	8.603
E_J/E_C	34.25	0.06	48.95	34.62

Table 8.2: Parameters of the transmon qubit characterized using the fundamental mode of JJAR. 2. 0. The theory parameters mentioned in the last two column are obtained from blackbox quantization simulations considering different superconducting gap for Aluminium (Δ) [118].

8.5 Conclusion

In conclusion, I have successfully engineered, fabricated and characterized a novel device for qubit readout. Due to the time constraints the pulse measurements and phase measurements on the qubit are currently on-going work in collaboration with my colleague. The device is engineered to have an anharmonicity of the JJA resonant modes approximately equal to the linewidth of a particular resonant modes of a JJA. Such that the bi-stability appears at very few photons. However, due to the influence of geometric inductance of Josephson junction thin films and anharmonicities of the device are reduced by a factor of 10 compared to the linewidth of a particular resonant modes. The future plans is to improve the coupling of a JJAR.2.0 to the waveguide by placing it at the center of the waveguide, and to reduce the geometric inductance of JJAR.2.0, hence should be able to utilize the engineered two mode system for good qubit readout.

Chapter 9

Conclusion

This experimental work in this thesis consists of three main accomplishments.

The first part was designing and building a 3D circuit QED setup from scratch in a new lab which involved the following main steps: producing high quality factor cavities; designing, building and installing the cryogenic microwave setup as well as the room temperature amplification chains.

The second part was the finite element numerical simulations, utilizing the naturally occuring dipolar interactions in 3D superconducting circuits to realize a platform for analogue quantum simulation of XY spin models. The possibility of realizing arbitrary lattice geometries with locally-tunable dipole moments [29], in combination with their large interaction strength, opens the door to the investigation of a series of phenomena in quantum magnetism in both 1D [121, 122] and 2D [106], complementing the remarkable developments in cold atom and trapped ion systems [96, 97, 98]. The idea discussed in chapter 5 are not limited to Transmon qubits, but could be implemented with, e.g., Xmon qubits [89] or Fluxonium qubits coupled to an antenna [123]. It would be interesting to explore these developments in view of realizing Hamiltonian dynamics for surface code architectures [8] or as a building block for coupled cavity array experiments [124, 125].

High Q resonators are important for a qubit readout. A design for MSRs with a low interface participation ratio embedded in a rectangular waveguide is engineered and characterized. The presented setup, is an ideal platform for implementing interacting spin systems [91, 29] where the MSR can be used for readout. In Fig. 9.1 shows a conceptual schematic for simulating spin chain physics. The orientation of the qubits



Figure 9.1: MSRs used for analog quantum simulation. Conceptual schematic of a rectangular waveguide setup using three MSRs as a readout for a chain of transmon qubits. The transmon qubits couple capacitively to each other to realize a system for analog quantum simulation of spin chain physics.

relative to the waveguide allows us to control the coupling of the qubits to the waveguide mode. Fig. 9.1 they are oriented along the axis of the waveguide which will lead to a large qubit-qubit interaction but negligible coupling to the waveguide. Three MSRs with different frequencies, all above the waveguide's cutoff, are used to read out selected qubits. Another interesting aspect of the setup mentioned in chapter 6 is the built-in protection from spontaneous emission due to the Purcell effect, similar to [74, 190] but broadband. Even though the qubit is strongly coupled to the resonator it can not decay through the resonator, as the waveguide acts as a filter if the qubit frequency is below the cutoff. This platform can also be used to investigate the interplay between short range direct interactions, long range photon mediated interaction via the waveguide [191] and dissipative coupling to an open system. It offers a new route to investigate nonequilibrium condensed matter problems and makes use of dissipative state engineering protocols to prepare many-body states and non-equilibrium phases [192, 193].

The third part consists of exploring physics in the mesoscopic regime using a JJAR. The device exhibits bi-stability at very few photons, this is achieved by engineering the Kerr interaction strength to be comparable to the linewidth. This proof of principle device demonstrates that it is possible for a few microwave photons in the readout mode to switch the photon occupation number by two orders of magnitude in the pump mode. As such it is a promising system to implement novel types of nondemolition measurements [194], single photon microwave transistors [195, 196], single photon microwave switches [197], Flip-Flop memories [198] or even elements for autonomous quantum error correction [154].

In the last chapter of this thesis, a modified JJAR.2.0 is designed and characterized

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for dispersive readout on the qubit. The principle idea of JJAR.2.0 is to exhibit bistability at a few photons. This can be achieved by engineering a device which will have anharmonicites equal to the linewidth of the resonant modes. The fundamental mode of the JJAR.2.0 is used to dispersively readout the state of a qubit with very few photons. However, the measured device has a geometric inductance $L_P \approx 0.1$ nH arising from the Josephson junctions and the connecting wires between the junctions is on the same order of the Josephson inductance $L_J \approx 0.755$ nH. Hence decreasing the anharmonicites by factors of 10 compared to the line-width of a particular resonant modes. Appendix A

Mathematica code :JJAR.2.0

```
98]:= ClearAll["Global`*"];
          c = 3 * 10^8;
          mu0 = 4 * Pi * 10^{(-7)};
          eps0 = 8.8 * 10^{(-12)};
          ele = 1.6 * 10^{(-19)};
          h = 6.626068 * 10^{(-34)};
          hBar = h / (2 * Pi);
          delta = 165 * ele * 10^{(-6)};
          kB = 1.3806503 * 10^{(-23)};
          T = 20 * 10^{(-3)}; (* temperature in K*)
          NN = 19; (*num juncs +1 *)
           \{L, \sigma L\} = \{0.755, 0.00\}; (* 10^{-9}, \sigma L is in fractional ie 0.03= 3\%;
          also, see below for "one messed up junction"*)
          Cs = N@85.0; (*10<sup>-15</sup>*) (*Sets ground mode freq*)
          Cj = N@30.0; (*10<sup>-15</sup>;*) (*Sets plasma freq with Lj*)
          C0 = 0.150; (* 10^{-15} Brings dispersion down *)
          Junc_\sigma = If[\sigmaL == 0, 1, RandomVariate[NormalDistribution[1, \sigmaL], NN - 1]];
          Ljs = N[Junc \sigma] * ConstantArray[L, NN - 1];
          Ljs[[1;;18]] = Ljs[[1;;18]] *1;
          LMi = DiagonalMatrix [Prepend[Ljs^(-1), 0]];
          CjM = DiagonalMatrix [Prepend [ConstantArray [Cj, NN - 1] / Junc_{\sigma}, 0]];
          MatrixForm[CjM];
          CsM = N[Normal[SparseArray[{i_, j_}] /; i = j \&\& i = 1 \rightarrow 1, \{NN, NN\}]] + 
                           ConstantArray[1, {NN, NN}]];
          CsM1 = N[Normal[SparseArray[{2, 2} \rightarrow 1, {NN, NN}]]];
          MatrixForm[CsM1];
          CsMid = N[Normal[SparseArray[\{10, 10\} \rightarrow 1, \{NN, NN\}]];
          MatrixForm[CsMid]
          CsM = CsM + 8.4 * CsMid;
          \texttt{COM} = \texttt{N}[\texttt{Normal}[\texttt{SparseArray}[\{\{\texttt{i},\texttt{j}\} / \texttt{;} \texttt{i} = \texttt{j} \& \texttt{i} = \texttt{1} \rightarrow \texttt{NN} - \texttt{2}, \texttt{i} = \texttt{N} \land \texttt{NN} - \texttt{NN} - \texttt{NN} = \texttt{N} \land 
                                    \{i_{, j}\} /; i \neq NN \& j \neq NN \rightarrow NN - Max[i, j]\}, \{NN, NN\}]]]; 
          MatrixForm[COM];
          COM = N[Normal[SparseArray[\{i_, j_\}] \rightarrow (NN+1) - Max[i, j], \{NN, NN\}]]];
          CM = (CjM + Cs * CsM + C0 * C0M) / 10^6;
          CMi = Inverse[CM];
           {evals, evecs} = N[Eigensystem[CMi.LMi]];
          If [Abs[evals[[-1]] + 0] > 10^{(-6)},
                  MessageDialog["Warning! The smallest eigenvalue was not 0!"]];
          evecs = Normalize /@evecs;
          evals = Reverse[evals]; evecs = Reverse[evecs];
          EF = evals; EV = evecs;
          evals = evals[[2;;-1]]; evecs = evecs[[2;;-1,2;;-1]];
          evecs = (If[#1[[1]] > 0, #1, -#1] &) /@evecs;
          freqs = Sqrt[evals] / (2 * Pi);
```

```
{freqs, evecs};
EjGHz[Lq_] := (hBar / (2ele))^2 / h / (Lq);
Norms = 2 * Diagonal [EV.LMi.EV<sup>T</sup>];
(*RMS norm*)Norms[[1]] = 1;
(*just to avoid avoid division by zero;
this value will still correspond to zero for the DC mode*)
PJs = (Prepend[Ljs^{-1}, 0] * #^2 \& @EV) / (Norms);
(*Matrix of PJs for each mode and for each junction*)
(*The "prepend 0" is there to deal with the DC mode,
which we will remove in the next line*)
\mathbb{P} = PJs[[2;; -1, 2;; -1]] (*get rid of DC mode and DC node with no JJ.*);
\mathbb{F} = \text{DiagonalMatrix} \left[ FR = Sqrt[EF[[2;;-1]]] / (2\pi) \right]
    (*undressed freqs, the same as abouve*) ];
Eji = DiagonalMatrix[(EjGHz /@Ljs)^-1];
X = F.P.Eji.P^{T}.F/2(*in GHz*);
\chi 1 = \{ \#, 10^{3} X [ [1, \#] ] \} \& /@ Range [8] ;
(*in MHz*)\chi3 = {#, 10^3 X[[3, #]]} & /@Range[8];
(*in MHz*)alpha = {#, 10^3 X[[#, #]]} & /@Range[8];
Print["Plasma frequency: Subscript[f, 0] = ", FR[[-1]], " GHz"];
Print["The first mode: Subscript[f, 1] = ", FR[[1]], " GHz"];
Print[
  "The anharmonicity of a junction alone (plasma mode): Subscript[\alpha, 0] = ",
  X[[8, 8]] * 10<sup>3</sup>, " MHz"];
Print["The anharmonicity of mode 1 (", FR[[1]],
   " GHz): Subscript[α, 1] = ", X[[1, 1]] * 10<sup>3</sup>, " MHz/photon ;"];
Print["The anharmonicity of mode 2 (", FR[[2]],
   " GHz): Subscript[\alpha, 2] = ", X[[2, 2]] * 10^3, " MHz/photon ;"];
Print["The anharmonicity of mode 2 (", FR[[3]],
   " GHz): Subscript[α, 2] = ", X[[3, 3]] * 10<sup>3</sup>, " MHz/photon ;"];
Print["The cross-Kerr of mode 1 and 1:Subscript[\chi, 1,2] = ",
  X[[1, 2]] * 10<sup>3</sup>, " MHz"];
ListPlot [{(*MeasFr,*)FR[[1;;10]]},
 PlotStyle → {Directive[PointSize[0.015], Darker[Red, 0.2]],
   Directive [PointSize [0.012], Blue] }, AspectRatio \rightarrow 1/1,
 ImageSize \rightarrow 300, Axes \rightarrow False, Frame \rightarrow True, FrameStyle \rightarrow Thick,
 LabelStyle \rightarrow Directive[FontSize \rightarrow 12, FontFamily \rightarrow "Arial", FontSlant \rightarrow "Plain"],
 FrameLabel \rightarrow {"mode #", "Frequency (GHz)"}
ListPlot [{alpha[[1;; 8]]},
 PlotStyle → {Directive[PointSize[0.015], Darker[Red, 0.2]],
   Directive[PointSize[0.012], Blue]}, AspectRatio -> 1/1,
 ImageSize \rightarrow 300, Axes \rightarrow False, Frame \rightarrow True, FrameStyle \rightarrow Thick,
```

```
LabelStyle \rightarrow Directive[FontSize \rightarrow 25,
```

1

```
FontFamily \rightarrow "Arial", FontSlant \rightarrow "Plain"],
```

```
FrameLabel \rightarrow {"mode #", "anharmonicity (\alpha) (MHz)"}
```

```
Plasma frequency: Subscript[f, 0] = 33.42 GHz
```

The first mode: Subscript[f, 1] = 5.94475 GHz

The anharmonicity of a junction alone (plasma mode): $Subscript[\alpha, 0] = 70.4634$ MHz The anharmonicity of mode 1 (5.94475 GHz): $Subscript[\alpha, 1] = 6.24752$ MHz/photon ; The anharmonicity of mode 2 (7.51134 GHz): $Subscript[\alpha, 2] = 7.35093$ MHz/photon ; The anharmonicity of mode 2 (31.0183 GHz): $Subscript[\alpha, 2] = 46.1776$ MHz/photon ; The cross-Kerr of mode 1 and 1: $Subscript[\chi, 1, 2] = 6.75565$ MHz



Appendix B

Fabrication recipe

Step	Description	Equipment used	Precision
1	Cleaving 4 inch	Diamond cutting tool	4-inch wafer is
	substrate		cleaved into 4 quarter's
2	Cleaning	Acetone	Ultrasonic bath for 15 mins
		Isopropanal	Ultrasonic bath for 10 mins
		Distilled water	Rinse in DI water
			followed by blow dry using N_2
3	Resist coating	MMA(8.5) MAA EL 13	Spin 40 s at 500 rpm,
		(copolymer)	Spin $60 \text{ s at } 1500 \text{ rpm}$
			Baking on hot plate at
			200° for 5 min
4	Resist coating	PMMA 950k	Spin 100 s at 2000 rpm
			Baking at 200° for 5 min
5	E-beam lithography	Reith 30 keV	Acceleration voltage 30 KV
			Big aperture:
			Beam current 5.5 nA
			Large area doses 360 $\mu C/cm^2$
			Small aperture:
			Beam current 36.5 pA
			Junction dose 600 $\mu C/cm^2$
			Undercut dose 160 $\mu C/cm^2$
6	Development	IPA:water 1:3	$105 \text{ s at } 6^{\circ}$
			5 s in DI water
			N_2 blow dry, low pressure
7	Al deposition	Plassys	$25 \text{ nm at } 1 \text{ nm/s and } 25^{\circ}$
		-MEB550S	
			Oxidation 10 mBar
			for 1 min
			$30 \text{ nm at } 1 \text{ nm/s amd } -25^{\circ}$
8	Lift-off	Acetone	4 hours at 60° C
			Rinse in IPA, DI
			Blow dry N_2

 Table B.1: Fabrication recipe for Josephson junctions.

Bibliography

- Eleanor Rieffel and Wolfgang Polak. Quantum computing: a gentle introduction. The MIT Press, 2011.
- [2] Peter W. Shor. Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer. SIAM Journal on Computing 0097-5397, 26(5):1484–1509, October 1997.
- [3] Lieven M. K. Vandersypen, Matthias Steffen, Gregory Breyta, Costantino S. Yannoni, Mark H. Sherwood, and Isaac L. Chuang. Experimental realization of Shor's quantum factoring algorithm using nuclear magnetic resonance. *Nature.*, 1476-4687, 414(6866):883–887, December 2001.
- [4] M. Gilowski, T. Wendrich, T. Müller, Ch. Jentsch, W. Ertmer, E. M. Rasel, and W. P. Schleich. Gauss sum factorization with cold atoms. *Phys. Rev. Lett.*, 100: 030201, Jan 2008.
- [5] Alberto Politi, Jonathan C. F. Matthews, and Jeremy L. O'Brien. Shor's quantum factoring algorithm on a photonic chip. *Science.*,0036-8075, 325(5945):1221–1221, 2009.
- [6] Thomas Monz, Daniel Nigg, Esteban A. Martinez, Matthias F. Brandl, Philipp Schindler, Richard Rines, Shannon X. Wang, Isaac L. Chuang, and Rainer Blatt. Realization of a scalable shor algorithm. *Science.*,0036-8075, 351(6277):1068–1070, 2016.
- [7] Erik Lucero, R. Barends, Y. Chen, J. Kelly, M. Mariantoni, A. Megrant, P. O'Malley, D. Sank, A. Vainsencher, J. Wenner, T. White, Y. Yin, A. N. Cleland, and John M. Martinis. Computing prime factors with a Josephson phase qubit quantum processor. *Nature Physics*, 8(10):719–723, October 2012.

- [8] Austin G. Fowler, Matteo Mariantoni, John M. Martinis, and Andrew N. Cleland. Surface codes: Towards practical large-scale quantum computation. *Phys. Rev. A*, 86:032324, Sep 2012.
- [9] Ryan Babbush, Jarrod McClean, Dave Wecker, Alán Aspuru-Guzik, and Nathan Wiebe. Chemical basis of trotter-suzuki errors in quantum chemistry simulation. *Phys. Rev. A*, 91:022311, Feb 2015.
- [10] Tadashi Kadowaki. Study of Optimization Problems by Quantum Annealing. PhD thesis, 1998.
- [11] Román Orús, Samuel Mugel, and Enrique Lizaso. Quantum computing for finance: Overview and prospects. *Reviews in Physics 2405-4283*, 4:100028, November 2019.
- [12] Javier Argüello-Luengo, Alejandro González-Tudela, Tao Shi, Peter Zoller, and J. Ignacio Cirac. Analogue quantum chemistry simulation. *Nature.*,0028-0836, 1476-4687, 574(7777):215-218, October 2019.
- Maria Schuld, Ilya Sinayskiy, and Francesco Petruccione. The quest for a Quantum Neural Network. *Quantum Information Processing*1573-1332, 13(11):2567-2586, November 2014.
- [14] Chih-Chun Chien, Michael Zwolak, and Massimiliano Di Ventra. Bosonic and fermionic transport phenomena of ultracold atoms in one-dimensional optical lattices. *Phys. Rev. A*, 85:041601, Apr 2012.
- [15] Daniel Greif, Thomas Uehlinger, Gregor Jotzu, Leticia Tarruell, and Tilman Esslinger. Short-Range Quantum Magnetism of Ultracold Fermions in an Optical Lattice. Science.,0036-8075, 1095-9203, 340(6138):1307-1310, June 2013.
- [16] Markus Greiner, Olaf Mandel, Tilman Esslinger, Theodor W. Hänsch, and Immanuel Bloch. Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms. *Nature.*, 1476-4687, 415(6867):39–44, January 2002.
- [17] Tim Byrnes, Na Young Kim, Kenichiro Kusudo, and Yoshihisa Yamamoto. Quantum simulation of fermi-hubbard models in semiconductor quantum-dot arrays. *Phys. Rev. B*, 78:075320, Aug 2008.
- [18] A. Friedenauer, H. Schmitz, J. T. Glueckert, D. Porras, and T. Schaetz. Simulating a quantum magnet with trapped ions. *Nature Physics*, 4:757–761, October 2008.

- [19] Y. Salathé, M. Mondal, M. Oppliger, J. Heinsoo, P. Kurpiers, A. Potočnik, A. Mezzacapo, U. Las Heras, L. Lamata, E. Solano, S. Filipp, and A. Wallraff. Digital quantum simulation of spin models with circuit quantum electrodynamics. *Phys. Rev. X*, 5:021027, Jun 2015.
- [20] B. D. Josephson. Possible new effects in superconductive tunnelling. *Physics Letters* ., 1(7):251–253, July 1962. ISSN 0031-9163.
- [21] Herbert Simanjuntak and Leon Gunther. Macroscopic quantum tunneling in unshunted Josephson junctions. Journal of Low Temperature Physics1573-7357, 78 (1):131–139, January 1990.
- [22] Alexandre Blais, Ren-Shou Huang, Andreas Wallraff, S. M. Girvin, and R. J. Schoelkopf. Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation. *Phys. Rev. A*, 69:062320, Jun 2004.
- [23] M. H. Devoret and R. J. Schoelkopf. Superconducting Circuits for Quantum Information: An Outlook. *Science.*,0036-8075, 1095-9203, 339(6124):1169–1174, March 2013.
- [24] Gary J. Mooney, Charles D. Hill, and Lloyd C. L. Hollenberg. Entanglement in a 20-Qubit Superconducting Quantum Computer. *Scientific Reports.*,2045-2322, 9 (1):1–8, September 2019.
- [25] Flora Graham. Daily briefing: Google's quantum-supremacy paper is here. Technical report, October 2019.
- [26] M. D. Reed, L. DiCarlo, S. E. Nigg, L. Sun, L. Frunzio, S. M. Girvin, and R. J. Schoelkopf. Realization of three-qubit quantum error correction with superconducting circuits. *Nature.*, 1476-4687, 482(7385):382–385, February 2012.
- [27] L. DiCarlo, J. M. Chow, J. M. Gambetta, Lev S. Bishop, B. R. Johnson, D. I. Schuster, J. Majer, A. Blais, L. Frunzio, S. M. Girvin, and R. J. Schoelkopf. Demonstration of two-qubit algorithms with a superconducting quantum processor. *Nature.*,1476-4687, 460(7252):240-244, July 2009.

- [28] Brian Vlastakis, Gerhard Kirchmair, Zaki Leghtas, Simon E. Nigg, Luigi Frunzio, S. M. Girvin, Mazyar Mirrahimi, M. H. Devoret, and R. J. Schoelkopf. Deterministically Encoding Quantum Information Using 100-Photon Schrödinger Cat States. *Science.*,0036-8075, 1095-9203, September 2013.
- [29] M. Dalmonte, S. I. Mirzaei, P. R. Muppalla, D. Marcos, P. Zoller, and G. Kirchmair. Realizing dipolar spin models with arrays of superconducting qubits. *Phys. Rev. B*, 92:174507, Nov 2015.
- [30] Hanhee Paik, D. I. Schuster, Lev S. Bishop, G. Kirchmair, G. Catelani, A. P. Sears, B. R. Johnson, M. J. Reagor, L. Frunzio, L. I. Glazman, S. M. Girvin, M. H. Devoret, and R. J. Schoelkopf. Observation of high coherence in josephson junction qubits measured in a three-dimensional circuit qed architecture. *Phys. Rev. Lett.*, 107:240501, Dec 2011.
- [31] Chanchal K. Majumdar and Dipan K. Ghosh. On Next-Nearest-Neighbor Interaction in Linear Chain. I. Journal of Mathematical Physics0022-2488, 10(8): 1388–1398, August 1969.
- [32] Christian Kraglund Andersen, Archana Kamal, Nicholas A. Masluk, Ioan M. Pop, Alexandre Blais, and Michel H. Devoret. Quantum versus classical switching dynamics of driven-dissipative kerr resonators. arXiv: 1906.10022, 2019.
- [33] R. Vijay, D. H. Slichter, and I. Siddiqi. Observation of quantum jumps in a superconducting artificial atom. *Phys. Rev. Lett.*, 106:110502, Mar 2011.
- [34] N. Bergeal, F. Schackert, M. Metcalfe, R. Vijay, V. E. Manucharyan, L. Frunzio, D. E. Prober, R. J. Schoelkopf, S. M. Girvin, and M. H. Devoret. Phasepreserving amplification near the quantum limit with a Josephson ring modulator. *Nature.*, 1476-4687, 465(7294):64–68, May 2010.
- [35] Patrick Winkel, Ivan Takmakov, Dennis Rieger, Luca Planat, Wiebke Hasch-Guichard, Lukas Grünhaupt, Nataliya Maleeva, Farshad Foroughi, Fabio Henriques, Kiril Borisov, Julian Ferrero, Alexey V. Ustinov, Wolfgang Wernsdorfer, Nicolas Roch, and Ioan M. Pop. Nondegenerate parametric amplifiers based on dispersion-engineered josephson-junction arrays. *Phys. Rev. Applied*, 13:024015, Feb 2020.

- [36] Ananda Roy and Michel Devoret. Introduction to parametric amplification of quantum signals with Josephson circuits. *Comptes Rendus Physique 1631-0705*, 17(7):740–755, August 2016.
- [37] S. Barzanjeh, M. Wulf, M. Peruzzo, M. Kalaee, P. B. Dieterle, O. Painter, and J. M. Fink. Mechanical on-chip microwave circulator. *Nature Communications2041-*1723, 8(1):1–7, October 2017.
- [38] R. Kautz, C. Hamilton, and F. Lloyd. Series-array josephson voltage standards. *IEEE Transactions on Magnetics* 1941-0069, 23, March 1987.
- [39] A. K. Jain, K. K. Likharev, J. E. Lukens, and J. E. Sauvageau. Mutual phaselocking in Josephson junction arrays. *Physics Reports.*,0370-1573, 109(6):309–426, July 1984.
- [40] M. A. Castellanos-Beltran and K. W. Lehnert. Widely tunable parametric amplifier based on a superconducting quantum interference device array resonator. *Applied Physics Letters 0003-6951*, 91(8):083509, August 2007.
- [41] P. Scarlino, D. J. van Woerkom, A. Stockklauser, J. V. Koski, M. C. Collodo, S. Gasparinetti, C. Reichl, W. Wegscheider, T. Ihn, K. Ensslin, and A. Wallraff. All-microwave control and dispersive readout of gate-defined quantum dot qubits in circuit quantum electrodynamics. *Phys. Rev. Lett.*, 122:206802, May 2019.
- [42] Gershon Kurizki, Patrice Bertet, Yuimaru Kubo, Klaus Mølmer, David Petrosyan, Peter Rabl, and Jörg Schmiedmayer. Quantum technologies with hybrid systems. Proceedings of the National Academy of Sciences0027-8424, 1091-6490, 112(13): 3866–3873, March 2015.
- [43] Nicholas A. Masluk, Ioan M. Pop, Archana Kamal, Zlatko K. Minev, and Michel H. Devoret. Microwave characterization of josephson junction arrays: Implementing a low loss superinductance. *Phys. Rev. Lett.*, 109:137002, Sep 2012.
- [44] P. R. Muppalla, O. Gargiulo, S. I. Mirzaei, B. Prasanna Venkatesh, M. L. Juan, L. Grünhaupt, I. M. Pop, and G. Kirchmair. Bistability in a mesoscopic josephson junction array resonator. *Phys. Rev. B*, 97:024518, Jan 2018.

- [45] I. Siddiqi, R. Vijay, F. Pierre, C. M. Wilson, M. Metcalfe, C. Rigetti, L. Frunzio, and M. H. Devoret. Rf-driven josephson bifurcation amplifier for quantum measurement. *Phys. Rev. Lett.*, 93:207002, Nov 2004.
- [46] Z. R. Lin, K. Inomata, K. Koshino, W. D. Oliver, Y. Nakamura, J. S. Tsai, and T. Yamamoto. Josephson parametric phase-locked oscillator and its application to dispersive readout of superconducting qubits. *Nature Communications*, 5(1):1–6, July 2014.
- [47] François Mallet, Florian R. Ong, Agustin Palacios-Laloy, François Nguyen, Patrice Bertet, Denis Vion, and Daniel Esteve. Single-shot qubit readout in circuit quantum electrodynamics. *Nature Physics1745-2481*, 5(11):791–795, November 2009.
- [48] A. Lupaşcu, S. Saito, T. Picot, P. C. de Groot, C. J. P. M. Harmans, and J. E. Mooij. Quantum non-demolition measurement of a superconducting two-level system. *Nature Physics1745-2481*, 3(2):119–123, February 2007.
- [49] F. R. Ong, M. Boissonneault, F. Mallet, A. Palacios-Laloy, A. Dewes, A. C. Doherty, A. Blais, P. Bertet, D. Vion, and D. Esteve. Circuit QED with a Nonlinear Resonator: Ac-Stark Shift and Dephasing. *Physical Review Letters*, 106(16): 167002, April 2011.
- [50] Jens Koch, Terri M. Yu, Jay Gambetta, A. A. Houck, D. I. Schuster, J. Majer, Alexandre Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf. Chargeinsensitive qubit design derived from the cooper pair box. *Phys. Rev. A*, 76: 042319, Oct 2007.
- [51] T. Weißl, B. Küng, E. Dumur, A. K. Feofanov, I. Matei, C. Naud, O. Buisson, F. W. J. Hekking, and W. Guichard. Kerr coefficients of plasma resonances in josephson junction chains. *Phys. Rev. B*, 92:104508, Sep 2015.
- [52] Yu. Krupko, V. D. Nguyen, T. Weißl, É. Dumur, J. Puertas, R. Dassonneville, C. Naud, F. W. J. Hekking, D. M. Basko, O. Buisson, N. Roch, and W. Hasch-Guichard. Kerr nonlinearity in a superconducting josephson metamaterial. *Phys. Rev. B*, 98:094516, Sep 2018.
- [53] David Zoepfl. Characterisation of stripline resonators in a waveguide. Master's thesis, University of Innsbruck, Innsbruck, 2017.

- [54] Lingzhen Guo, Zhigang Zheng, Xin-Qi Li, and YiJing Yan. Dynamic quantum tunneling in mesoscopic driven duffing oscillators. *Phys. Rev. E*, 84:011144, Jul 2011.
- [55] Lingzhen Guo, Vittorio Peano, M. Marthaler, and M. I. Dykman. Quantum critical temperature of a modulated oscillator. *Phys. Rev. A*, 87:062117, Jun 2013.
- [56] David M. Pozar. Microwave Engineering. Wiley publisher, Hoboken, NJ, 4th edition, 2012.
- [57] T. Brecht et al. Demonstration of superconducting micromachined cavities. Applied Physics Letters, 107, Nov 2015.
- [58] Jiansong Gao. The Physics of Superconducting Microwave Resonators. PhD thesis, California Institute of Technology, 2008.
- [59] Matthew Reagor, Hanhee Paik, Gianluigi Catelani, Luyan Sun, Christopher Axline, Eric Holland, Ioan M. Pop, Nicholas A. Masluk, Teresa Brecht, Luigi Frunzio, Michel H. Devoret, Leonid Glazman, and Robert J. Schoelkopf. Reaching 10ms single photon lifetimes for superconducting aluminum cavities. *Applied Physics Letters 0003-6951*, 102(19):192604, May 2013.
- [60] Sidney Shapiro. Josephson currents in superconducting tunneling: The effect of microwaves and other observations. *Phys. Rev. Lett.*, 11:80–82, Jul 1963.
- [61] Vinay Ambegaokar and Alexis Baratoff. Tunneling between superconductors. Phys. Rev. Lett., 10:486–489, Jun 1963.
- [62] Thomas R. Lemberger, Aaron A. Pesetski, and Stefan J. Turneaure. Effect of thermal phase fluctuations on the inductances of josephson junctions, arrays of junctions, and superconducting films. *Phys. Rev. B*, 61:1483–1489, Jan 2000.
- [63] T. Weißl, G. Rastelli, I. Matei, I. M. Pop, O. Buisson, F. W. J. Hekking, and W. Guichard. Bloch band dynamics of a josephson junction in an inductive environment. *Phys. Rev. B*, 91:014507, Jan 2015.
- [64] K. A. Matveev, A. I. Larkin, and L. I. Glazman. Persistent current in superconducting nanorings. *Phys. Rev. Lett.*, 89:096802, Aug 2002.

- [65] A. A. Golubov, M. Yu. Kupriyanov, and E. Il'ichev. The current-phase relation in josephson junctions. *Rev. Mod. Phys.*, 76:411–469, Apr 2004.
- [66] Peter Agren. Charging effects in small capacitance Josephson junction circuits. PhD thesis, 2002.
- [67] Ioan Mihai Pop. Sauts quantiques de phase dans des réseaux de jonctions Josephson. PhD thesis, 2011.
- [68] B. Lee Roberts. Notes on linear and nonlinear oscillators and periodic waves. 2011.
- [69] R. E. Mickens. Oscillations in planar Dynamics systems. World scientific, 1996.
- [70] A. H. Nayfeh and D. T. Mook. Non-linear oscillations. Wiley-Interscience, Newyork, 1979.
- [71] Peter Hänggi, Peter Talkner, and Michal Borkovec. Reaction-rate theory: fifty years after kramers. *Rev. Mod. Phys.*, 62:251–341, Apr 1990.
- [72] G. P. Agrawal and H. J. Carmichael. Optical bistability through nonlinear dispersion and absorption. *Phys. Rev. A*, 19:2074–2086, May 1979.
- [73] F. R. Ong, M. Boissonneault, F. Mallet, A. Palacios-Laloy, A. Dewes, A. C. Doherty, A. Blais, P. Bertet, D. Vion, and D. Esteve. Circuit qed with a nonlinear resonator: ac-stark shift and dephasing. *Phys. Rev. Lett.*, 106:167002, Apr 2011.
- [74] M. D. Reed, B. R. Johnson, A. A. Houck, L. DiCarlo, J. M. Chow, D. I. Schuster, L. Frunzio, and R. J. Schoelkopf. Fast reset and suppressing spontaneous emission of a superconducting qubit. *Applied Physics Letters* 0003-6951, 96(20):203110, May 2010.
- [75] Triton 400 dilution refrigerator. URL www.oxinst.com.
- [76] J. W. Ekin. Experimental techniques for low-temperature measurements: cryostat design, material properties, and superconductor critical-current testing. Oxford University Press, Oxford; New York, 2006.
- [77] J. B. Johnson. Thermal agitation of electricity in conductors. Phys. Rev., 32: 97–109, Jul 1928.

- [78] Heat load on different stages of a cryostat due to stainless steel cables. URL http://cryogenics.nist.gov/MPropsMAY/materialproperties.htm.
- [79] Dongmei Bi, Huanxin Chen, and Tian Ye. Influences of temperature and contact pressure on thermal contact resistance at interfaces at cryogenic temperatures. *Cryogenics 0011-2275*, 52(7):403–409, July 2012.
- [80] Anthony C. Crawford. Mechanically Stimulated Domain Re-arrangement in Cryogenic Magnetic Shielding Materials. arXiv:1601.00592 [physics], January 2016.
- [81] E. Hyyppä, M. Jenei, S. Masuda, V. Sevriuk, K. Y. Tan, M. Silveri, J. Goetz, M. Partanen, R. E. Lake, L. Grönberg, and M. Möttönen. Calibration of cryogenic amplification chains using normal-metal-insulator-superconductor junctions. *Applied Physics Letters 0003-6951*, 114(19):192603, May 2019.
- [82] Florent Lecocq, Ioan M. Pop, Zhihui Peng, Iulian Matei, Thierry Crozes, Thierry Fournier, Cécile Naud, Wiebke Guichard, and Olivier Buisson. Junction fabrication by shadow evaporation without a suspended bridge. Nanotechnology 0957-4484, 22(31):315302, July 2011.
- [83] G. Dolan and J. Dunsmuir. Very small (20 nm) lithographic wires, dots, rings, and tunnel junctions. *Science direct*, 152:7–13, AUG 1988.
- [84] R. L. Stow. Titanium as a Gettering Material. Nature., 1476-4687, 184(4685):
 542–543, August 1959.
- [85] A. Dewes, F. R. Ong, V. Schmitt, R. Lauro, N. Boulant, P. Bertet, D. Vion, and D. Esteve. Characterization of a two-transmon processor with individual singleshot qubit readout. *Phys. Rev. Lett.*, 108:057002, Feb 2012.
- [86] Arjan F. van Loo, Arkady Fedorov, Kevin Lalumière, Barry C. Sanders, Alexandre Blais, and Andreas Wallraff. Photon-Mediated Interactions Between Distant Artificial Atoms. *Science0036-8075*, 1095-9203, 342(6165):1494–1496, December 2013.
- [87] D. Ristè, M. Dukalski, C. A. Watson, G. de Lange, M. J. Tiggelman, Ya M. Blanter, K. W. Lehnert, R. N. Schouten, and L. DiCarlo. Deterministic entanglement of superconducting qubits by parity measurement and feedback. *Nature.*,1476-4687, 502(7471):350-354, October 2013.

- [88] N. Roch, M. E. Schwartz, F. Motzoi, C. Macklin, R. Vijay, A. W. Eddins, A. N. Korotkov, K. B. Whaley, M. Sarovar, and I. Siddiqi. Observation of measurement-induced entanglement and quantum trajectories of remote superconducting qubits. *Phys. Rev. Lett.*, 112:170501, Apr 2014.
- [89] R. Barends, J. Kelly, A. Megrant, A. Veitia, D. Sank, E. Jeffrey, T. C. White, J. Mutus, A. G. Fowler, B. Campbell, Y. Chen, Z. Chen, B. Chiaro, A. Dunsworth, C. Neill, P. O'Malley, P. Roushan, A. Vainsencher, J. Wenner, A. N. Korotkov, A. N. Cleland, and John M. Martinis. Superconducting quantum circuits at the surface code threshold for fault tolerance. *Nature.*,1476-4687, 508(7497):500–503, April 2014.
- [90] Andrew A. Houck, Hakan E. Türeci, and Jens Koch. On-chip quantum simulation with superconducting circuits. *Nature Physics* 1745-2481, 8(4):292–299, April 2012.
- [91] Oliver Viehmann, Jan von Delft, and Florian Marquardt. Observing the nonequilibrium dynamics of the quantum transverse-field ising chain in circuit qed. *Phys. Rev. Lett.*, 110:030601, Jan 2013.
- [92] Oliver Viehmann, Jan von Delft, and Florian Marquardt. The quantum transversefield Ising chain in circuit quantum electrodynamics: effects of disorder on the nonequilibrium dynamics. New Journal of Physics1367-2630, 15(3):035013, March 2013.
- [93] A. Mezzacapo, L. Lamata, S. Filipp, and E. Solano. Many-body interactions with tunable-coupling transmon qubits. *Phys. Rev. Lett.*, 113:050501, Jul 2014.
- [94] Uta Naether, Fernando Quijandría, Juan José García-Ripoll, and David Zueco. Stationary discrete solitons in a driven dissipative bose-hubbard chain. *Phys. Rev.* A, 91:033823, Mar 2015.
- [95] Chad Rigetti, Jay M. Gambetta, Stefano Poletto, B. L. T. Plourde, Jerry M. Chow, A. D. Córcoles, John A. Smolin, Seth T. Merkel, J. R. Rozen, George A. Keefe, Mary B. Rothwell, Mark B. Ketchen, and M. Steffen. Superconducting qubit in a waveguide cavity with a coherence time approaching 0.1 ms. *Phys. Rev. B*, 86:100506, Sep 2012.
- [96] Immanuel Bloch, Jean Dalibard, and Sylvain Nascimbène. Quantum simulations with ultracold quantum gases. *Nature Physics*1745-2481, 8(4):267–276, April 2012.

- [97] R. Blatt and C. F. Roos. Quantum simulations with trapped ions. Nature Physics1745-2481, 8(4):277-284, April 2012.
- [98] I. M. Georgescu, S. Ashhab, and Franco Nori. Quantum simulation. Rev. Mod. Phys., 86:153–185, Mar 2014.
- [99] Anatoli Polkovnikov, Krishnendu Sengupta, Alessandro Silva, and Mukund Vengalattore. Nonequilibrium dynamics of closed interacting quantum systems. *Rev.Mod.Phys.*, 83:863, 2011.
- [100] C Lacroix, P Mendels, and F Mila. Introduction to Frustrated Magnetism: Materials, Experiments, Theory. Springer Series in Solid-State Sciences. Springer, Berlin, Heidelberg, 2011.
- [101] Alexander O. Gogolin, Alexander A. Nersesyan, and Alexei M. Tsvelik. Bosonization and Strongly Correlated Systems. Cambridge University Press, 2004.
- [102] Thierry Giamarchi. Quantum Physics in One Dimension. Oxford University Press, 2003.
- [103] Tapan Mishra, Ramesh V. Pai, Subroto Mukerjee, and Arun Paramekanti. Quantum phases and phase transitions of frustrated hard-core bosons on a triangular ladder. *Phys. Rev. B*, 87:174504, May 2013.
- [104] N. D. Mermin and H. Wagner. Absence of ferromagnetism or antiferromagnetism in one- or two-dimensional isotropic heisenberg models. *Phys. Rev. Lett.*, 17:1307– 1307, Dec 1966.
- [105] P. C. Hohenberg. Existence of long-range order in one and two dimensions. *Phys. Rev.*, 158:383–386, Jun 1967.
- [106] D. Peter, S. Müller, S. Wessel, and H. P. Büchler. Anomalous behavior of spin systems with dipolar interactions. *Phys. Rev. Lett.*, 109:025303, Jul 2012.
- [107] S. Hacohen-Gourgy, V. V. Ramasesh, C. De Grandi, I. Siddiqi, and S. M. Girvin. Cooling and autonomous feedback in a bose-hubbard chain with attractive interactions. *Phys. Rev. Lett.*, 115:240501, Dec 2015.
- [108] JD Jackson. Classical electrodynamics. Wiley, 1998.
- [109] Ansys. HFSS (Version 13). www.ansys.com.

- [110] A. P. Sears, A. Petrenko, G. Catelani, L. Sun, Hanhee Paik, G. Kirchmair, L. Frunzio, L. I. Glazman, S. M. Girvin, and R. J. Schoelkopf. Photon shot noise dephasing in the strong-dispersive limit of circuit qed. *Phys. Rev. B*, 86:180504, Nov 2012.
- [111] W. E. Shanks, D. L. Underwood, and A. A. Houck. A scanning transmon qubit for strong coupling circuit quantum electrodynamics. *Nature Communications2041-1723*, 4(1):1–6, June 2013.
- [112] M. Hatridge, S. Shankar, M. Mirrahimi, F. Schackert, K. Geerlings, T. Brecht, K. M. Sliwa, B. Abdo, L. Frunzio, S. M. Girvin, R. J. Schoelkopf, and M. H. Devoret. Quantum Back-Action of an Individual Variable-Strength Measurement. *Science.*,0036-8075, 1095-9203, 339(6116):178–181, January 2013.
- [113] Oliver Gywat, Florian Meier, Daniel Loss, and David D. Awschalom. Dynamics of coupled qubits interacting with an off-resonant cavity. *Phys. Rev. B*, 73:125336, Mar 2006.
- [114] Blake R. Johnson. Controlling Photons in Superconducting Electrical Circuits. PhD thesis, Yale University, 2010.
- [115] O. Gargiulo, S. Oleschko, J. Prat-Camps, M. Zanner, and G. Kirchmair. Fast flux control of 3d transmon qubits using a magnetic hose. arXiv: 1811.10875, 2018.
- [116] J. D. Strand, Matthew Ware, Félix Beaudoin, T. A. Ohki, B. R. Johnson, Alexandre Blais, and B. L. T. Plourde. First-order sideband transitions with flux-driven asymmetric transmon qubits. *Phys. Rev. B*, 87:220505, Jun 2013.
- [117] Liang-Hui Du, J. Q. You, and Lin Tian. Superconducting circuit probe for analog quantum simulators. *Phys. Rev. A*, 92:012330, Jul 2015.
- [118] Simon E. Nigg, Hanhee Paik, Brian Vlastakis, Gerhard Kirchmair, S. Shankar, Luigi Frunzio, M. H. Devoret, R. J. Schoelkopf, and S. M. Girvin. Black-box superconducting circuit quantization. *Phys. Rev. Lett.*, 108:240502, Jun 2012.
- [119] Firat Solgun, David W. Abraham, and David P. DiVincenzo. Blackbox quantization of superconducting circuits using exact impedance synthesis. *Phys. Rev. B*, 90:134504, Oct 2014.
- [120] John Martinis et al. Calculation of coupling capacitance in planar electrodes. arXiv:1410.3458v1, 2014.

- [121] R. A. Hyman, Kun Yang, R. N. Bhatt, and S. M. Girvin. Random bonds and topological stability in gapped quantum spin chains. *Phys. Rev. Lett.*, 76:839–842, Jan 1996.
- [122] Armin Rahmani, Adrian E. Feiguin, and Cristian D. Batista. Anyonic liquids in nearly saturated spin chains. *Phys. Rev. Lett.*, 113:267201, Dec 2014.
- [123] U. Vool, I. M. Pop, K. Sliwa, B. Abdo, C. Wang, T. Brecht, Y. Y. Gao, S. Shankar, M. Hatridge, G. Catelani, M. Mirrahimi, L. Frunzio, R. J. Schoelkopf, L. I. Glazman, and M. H. Devoret. Non-poissonian quantum jumps of a fluxonium qubit due to quasiparticle excitations. *Phys. Rev. Lett.*, 113:247001, Dec 2014.
- [124] Michael J. Hartmann, Fernando G. S. L. Brandão, and Martin B. Plenio. Strongly interacting polaritons in coupled arrays of cavities. *Nature Physics1745-2481*, 2 (12):849–855, December 2006.
- [125] Dario Gerace, Hakan E. Türeci, Atac Imamoglu, Vittorio Giovannetti, and Rosario Fazio. The quantum-optical Josephson interferometer. *Nature Physics.*,1745-2481, 5(4):281–284, April 2009.
- [126] Jay Gambetta, Alexandre Blais, M. Boissonneault, A. A. Houck, D. I. Schuster, and S. M. Girvin. Quantum trajectory approach to circuit qed: Quantum jumps and the zeno effect. *Phys. Rev. A*, 77:012112, Jan 2008.
- [127] C. Axline, M. Reagor, R. Heeres, P. Reinhold, C. Wang, K. Shain, W. Pfaff, Y. Chu, L. Frunzio, and R. J. Schoelkopf. An architecture for integrating planar and 3D cQED devices. *Applied Physics Letters* . 0003-6951, 109(4):042601, July 2016.
- [128] J. Majer, J. M. Chow, J. M. Gambetta, Jens Koch, B. R. Johnson, J. A. Schreier,
 L. Frunzio, D. I. Schuster, A. A. Houck, A. Wallraff, A. Blais, M. H. Devoret,
 S. M. Girvin, and R. J. Schoelkopf. Coupling superconducting qubits via a cavity
 bus. *Nature.*, 1476-4687, 449(7161):443-447, September 2007.
- [129] D. P. Pappas, M. R. Vissers, D. S. Wisbey, J. S. Kline, and J. Gao. Two Level System Loss in Superconducting Microwave Resonators. *IEEE Transactions on Applied Superconductivity 1051-8223*, 21(3):871–874, June 2011.

- [130] Jiansong Gao, Miguel Daal, Anastasios Vayonakis, Shwetank Kumar, Jonas Zmuidzinas, Bernard Sadoulet, Benjamin A. Mazin, Peter K. Day, and Henry G. Leduc. Experimental evidence for a surface distribution of two-level systems in superconducting lithographed microwave resonators. *Applied Physics Letters 0003-6951*, 92(15):152505, April 2008.
- [131] R. Barends, N. Vercruyssen, A. Endo, P. J. de Visser, T. Zijlstra, T. M. Klapwijk, P. Diener, S. J. C. Yates, and J. J. A. Baselmans. Minimal resonator loss for circuit quantum electrodynamics. *Applied Physics Letters 0003-6951*, 97(2):023508, July 2010.
- [132] K. Geerlings, S. Shankar, E. Edwards, L. Frunzio, R. J. Schoelkopf, and M. H. Devoret. Improving the quality factor of microwave compact resonators by optimizing their geometrical parameters. *Applied Physics Letters 0003-6951*, 100(19): 192601, May 2012.
- [133] Y. Chu, C. Axline, C. Wang, T. Brecht, Y. Y. Gao, L. Frunzio, and R. J. Schoelkopf. Suspending superconducting qubits by silicon micromachining. *Applied Physics Letters 0003-6951*, 109(11):112601, September 2016.
- [134] C. Wang, C. Axline, Y. Y. Gao, T. Brecht, Y. Chu, L. Frunzio, M. H. Devoret, and R. J. Schoelkopf. Surface participation and dielectric loss in superconducting qubits. *Applied Physics Letters 0003-6951*, 107(16):162601, October 2015.
- [135] A. Bruno, G. de Lange, S. Asaad, K. L. van der Enden, N. K. Langford, and L. Di-Carlo. Reducing intrinsic loss in superconducting resonators by surface treatment and deep etching of silicon substrates. *Applied Physics Letters 0003-6951*, 106(18): 182601, May 2015.
- [136] A. Megrant, C. Neill, R. Barends, B. Chiaro, Yu Chen, L. Feigl, J. Kelly, Erik Lucero, Matteo Mariantoni, P. J. J. O'Malley, D. Sank, A. Vainsencher, J. Wenner, T. C. White, Y. Yin, J. Zhao, C. J. Palmstrøm, John M. Martinis, and A. N. Cleland. Planar superconducting resonators with internal quality factors above one million. *Applied Physics Letters 0003-6951*, 100(11):113510, March 2012.
- [137] T. Brecht, M. Reagor, Y. Chu, W. Pfaff, C. Wang, L. Frunzio, M. H. Devoret, and R. J. Schoelkopf. Demonstration of superconducting micromachined cavities. *Applied Physics Letters 0003-6951*, 107(19):192603, November 2015.

- [138] J. Wenner, M. Neeley, Radoslaw C. Bialczak, M. Lenander, Erik Lucero, A. D. O'Connell, D. Sank, H. Wang, M. Weides, A. N. Cleland, and John M. Martinis. Wirebond crosstalk and cavity modes in large chip mounts for superconducting qubits. Superconductor Science and Technology 0953-2048, 24(6):065001, March 2011.
- [139] Zijun Chen, A. Megrant, J. Kelly, R. Barends, J. Bochmann, Yu Chen, B. Chiaro, A. Dunsworth, E. Jeffrey, J. Y. Mutus, P. J. J. O'Malley, C. Neill, P. Roushan, D. Sank, A. Vainsencher, J. Wenner, T. C. White, A. N. Cleland, and John M. Martinis. Fabrication and characterization of aluminum airbridges for superconducting microwave circuits. *Applied Physics Letters* 0003-6951, 104(5):052602, February 2014.
- [140] S. Probst, F. B. Song, P. A. Bushev, A. V. Ustinov, and M. Weides. Efficient and robust analysis of complex scattering data under noise in microwave resonators. *Review of Scientific Instruments 0034-6748*, 86(2):024706, February 2015.
- [141] C. Song, T. W. Heitmann, M. P. DeFeo, K. Yu, R. McDermott, M. Neeley, John M. Martinis, and B. L. T. Plourde. Microwave response of vortices in superconducting thin films of re and al. *Phys. Rev. B*, 79:174512, May 2009.
- [142] D. Janjušević, M. S. Grbić, M. Požek, A. Dulčić, D. Paar, B. Nebendahl, and T. Wagner. Microwave response of thin niobium films under perpendicular static magnetic fields. *Phys. Rev. B*, 74:104501, Sep 2006.
- [143] Temperature Dependence of the Surface Resistance of Niobium at 1300 MHz -Comparison to BCS Theory -, 1999. Proceedings of the 1999 Workshop on RF Superconductivity.
- [144] Rudolf Gross and Achim Marx. Festkörperphysik. De Gruyter Oldenbourg, Berlin, Boston, 2., akt. aufl. edition, 2014.
- [145] Conductivity and Resistivity Values for Aluminum & Alloys. Collaboration for NDT Education, 2002.
- [146] M. S. Khalil, M. J. A. Stoutimore, F. C. Wellstood, and K. D. Osborn. An analysis method for asymmetric resonator transmission applied to superconducting devices. *Journal of Applied Physics 0021-8979*, 111(5):054510, March 2012.

- [147] D. C. Mattis and J. Bardeen. Theory of the anomalous skin effect in normal and superconducting metals. *Phys. Rev.*, 111:412–417, Jul 1958.
- [148] J. Kelly, R. Barends, A. G. Fowler, A. Megrant, E. Jeffrey, T. C. White, D. Sank, J. Y. Mutus, B. Campbell, Yu Chen, Z. Chen, B. Chiaro, A. Dunsworth, I.-C. Hoi, C. Neill, P. J. J. O'Malley, C. Quintana, P. Roushan, A. Vainsencher, J. Wenner, A. N. Cleland, and John M. Martinis. State preservation by repetitive error detection in a superconducting quantum circuit. *Nature.*,1476-4687, 519 (7541):66-69, March 2015.
- [149] A. D. Córcoles, Easwar Magesan, Srikanth J. Srinivasan, Andrew W. Cross, M. Steffen, Jay M. Gambetta, and Jerry M. Chow. Demonstration of a quantum error detection code using a square lattice of four superconducting qubits. *Nature Communications2041-1723*, 6(1):1–10, April 2015.
- [150] L. Sun, A. Petrenko, Z. Leghtas, B. Vlastakis, G. Kirchmair, K. M. Sliwa, A. Narla, M. Hatridge, S. Shankar, J. Blumoff, L. Frunzio, M. Mirrahimi, M. H. Devoret, and R. J. Schoelkopf. Tracking photon jumps with repeated quantum non-demolition parity measurements. *Nature.*, 1476-4687, 511(7510):444-448, July 2014.
- [151] Chen Wang, Yvonne Y. Gao, Philip Reinhold, R. W. Heeres, Nissim Ofek, Kevin Chou, Christopher Axline, Matthew Reagor, Jacob Blumoff, K. M. Sliwa, L. Frunzio, S. M. Girvin, Liang Jiang, M. Mirrahimi, M. H. Devoret, and R. J. Schoelkopf. A Schrödinger cat living in two boxes. *Science.*,0036-8075, 1095-9203, 352(6289): 1087–1091, May 2016.
- [152] N. Bergeal, R. Vijay, V. E. Manucharyan, I. Siddiqi, R. J. Schoelkopf, S. M. Girvin, and M. H. Devoret. Analog information processing at the quantum limit with a Josephson ring modulator. *Nature Physics1745-2481*, 6(4):296–302, April 2010.
- [153] Hideo Mabuchi. Cavity-qed models of switches for attojoule-scale nanophotonic logic. Phys. Rev. A, 80:045802, Oct 2009.
- [154] Joseph Kerckhoff, Hendra I. Nurdin, Dmitri S. Pavlichin, and Hideo Mabuchi. Designing quantum memories with embedded control: Photonic circuits for autonomous quantum error correction. *Phys. Rev. Lett.*, 105:040502, Jul 2010.

- [155] Ranojoy Bose, Deepak Sridharan, Hyochul Kim, Glenn S. Solomon, and Edo Waks. Low-photon-number optical switching with a single quantum dot coupled to a photonic crystal cavity. *Phys. Rev. Lett.*, 108:227402, May 2012.
- [156] Wenlan Chen, Kristin M. Beck, Robert Bücker, Michael Gullans, Mikhail D. Lukin, Haruka Tanji-Suzuki, and Vladan Vuletić. All-Optical Switch and Transistor Gated by One Stored Photon. *Science.*,0036-8075, 1095-9203, 341(6147): 768–770, August 2013.
- [157] Vladimir E. Manucharyan, Jens Koch, Leonid I. Glazman, and Michel H. Devoret. Fluxonium: Single Cooper-Pair Circuit Free of Charge Offsets. *Science.*,0036-8075, 1095-9203, 326(5949):113–116, October 2009.
- [158] Ioan M. Pop, Kurtis Geerlings, Gianluigi Catelani, Robert J. Schoelkopf, Leonid I. Glazman, and Michel H. Devoret. Coherent suppression of electromagnetic dissipation due to superconducting quasiparticles. *Nature.*,1476-4687, 508(7496): 369–372, April 2014.
- [159] M. T. Bell, W. Zhang, L. B. Ioffe, and M. E. Gershenson. Spectroscopic evidence of the aharonov-casher effect in a cooper pair box. *Phys. Rev. Lett.*, 116:107002, Mar 2016.
- [160] C. Macklin, K. O'Brien, D. Hover, M. E. Schwartz, V. Bolkhovsky, X. Zhang, W. D. Oliver, and I. Siddiqi. A near-quantum-limited Josephson traveling-wave parametric amplifier. *Science.*, 0036-8075, 1095-9203, 350(6258):307-310, October 2015.
- [161] M. A. Castellanos-Beltran, K. D. Irwin, G. C. Hilton, L. R. Vale, and K. W. Lehnert. Amplification and squeezing of quantum noise with a tunable Josephson metamaterial. *Nature Physics* 1745-2481, 4(12):929–931, December 2008.
- [162] Christopher Eichler and Andreas Wallraff. Controlling the dynamic range of a Josephson parametric amplifier. EPJ Quantum Technology 2196-0763, 1(1):2, January 2014.
- [163] Carsten Hutter, Erik A. Tholén, Kai Stannigel, Jack Lidmar, and David B. Haviland. Josephson junction transmission lines as tunable artificial crystals. *Phys. Rev. B*, 83:014511, Jan 2011.

- [164] J. Bourassa, F. Beaudoin, Jay M. Gambetta, and A. Blais. Josephson-junctionembedded transmission-line resonators: From kerr medium to in-line transmon. *Phys. Rev. A*, 86:013814, Jul 2012.
- [165] M. Marthaler and M. I. Dykman. Switching via quantum activation: A parametrically modulated oscillator. *Phys. Rev. A*, 73:042108, Apr 2006.
- [166] P. L. García-Müller, F. Borondo, Rigoberto Hernandez, and R. M. Benito. Solventinduced acceleration of the rate of activation of a molecular reaction. *Phys. Rev. Lett.*, 101:178302, Oct 2008.
- [167] Mark D. McDonnell and Lawrence M. Ward. The benefits of noise in neural systems: bridging theory and experiment. *Nature Reviews Neuroscience1471-0048*, 12(7):415-425, July 2011.
- [168] V. M. Eguíluz, M. Ospeck, Y. Choe, A. J. Hudspeth, and M. O. Magnasco. Essential nonlinearities in hearing. *Phys. Rev. Lett.*, 84:5232–5235, May 2000. doi: 10.1103/PhysRevLett.84.5232.
- [169] Paolo Silvestrini, Sergio Pagano, Roberto Cristiano, O. Liengme, and K. E. Gray. Effect of dissipation on thermal activation in an underdamped josephson junction: First evidence of a transition between different damping regimes. *Phys. Rev. Lett.*, 60:844–847, Feb 1988.
- [170] Emmanuel Turlot, Daniel Esteve, Cristian Urbina, John M. Martinis, Michel H. Devoret, Sebastian Linkwitz, and Hermann Grabert. Escape oscillations of a josephson junction switching out of the zero-voltage state. *Phys. Rev. Lett.*, 62: 1788–1791, Apr 1989.
- [171] Th. K. Mavrogordatos, G. Tancredi, M. Elliott, M. J. Peterer, A. Patterson, J. Rahamim, P. J. Leek, E. Ginossar, and M. H. Szymańska. Simultaneous bistability of a qubit and resonator in circuit quantum electrodynamics. *Phys. Rev. Lett.*, 118:040402, Jan 2017.
- [172] F. Ricci, R. A. Rica, M. Spasenović, J. Gieseler, L. Rondin, L. Novotny, and R. Quidant. Optically levitated nanoparticle as a model system for stochastic bistable dynamics. *Nature Communications2041-1723*, 8(1):1–7, May 2017.

- [173] Loïc Rondin, Jan Gieseler, Francesco Ricci, Romain Quidant, Christoph Dellago, and Lukas Novotny. Direct measurement of Kramers turnover with a levitated nanoparticle. *Nature Nanotechnology* 1748-3395, 12(12):1130–1133, December 2017.
- [174] Florent Lecocq, Ioan M. Pop, Zhihui Peng, Iulian Matei, Thierry Crozes, Thierry Fournier, Cécile Naud, Wiebke Guichard, and Olivier Buisson. Junction fabrication by shadow evaporation without a suspended bridge. Nanotechnology 0957-4484, 22(31):315302, July 2011.
- [175] I. M. Pop, I. Protopopov, F. Lecocq, Z. Peng, B. Pannetier, O. Buisson, and W. Guichard. Measurement of the effect of quantum phase slips in a Josephson junction chain. *Nature Physics1745-2481*, 6(8):589–592, August 2010.
- [176] A. Kou, W. C. Smith, U. Vool, I. M. Pop, K. M. Sliwa, M. H. Hatridge, L. Frunzio, and M. H. Devoret. Simultaneous monitoring of fluxonium qubits in a waveguide. *arxiv:* 1705.05712.
- [177] S. Probst, F. B. Song, P. A. Bushev, A. V. Ustinov, and M. Weides. Efficient and robust analysis of complex scattering data under noise in microwave resonators. *Review of Scientific Instruments* 0034-6748, 86(2):024706, February 2015.
- [178] Peter Hänggi, Peter Talkner, and Michal Borkovec. Reaction-rate theory: fifty years after kramers. *Rev. Mod. Phys.*, 62:251–341, Apr 1990.
- [179] G. P. Agrawal and H. J. Carmichael. Optical bistability through nonlinear dispersion and absorption. *Phys. Rev. A*, 19:2074–2086, May 1979.
- [180] D. G. Luchinsky, R. Mannella, P. V. E. McClintock, M. I. Dykman, and V. N. Smelyanskiy. Thermally activated escape of driven systems: the activation energy. *Journal of Physics A: Mathematical and General0305-4470*, 32(27):L321–L327, January 1999.
- [181] K. Geerlings, Z. Leghtas, I. M. Pop, S. Shankar, L. Frunzio, R. J. Schoelkopf, M. Mirrahimi, and M. H. Devoret. Demonstrating a driven reset protocol for a superconducting qubit. *Phys. Rev. Lett.*, 110:120501, Mar 2013.
- [182] X. Y. Jin, A. Kamal, A. P. Sears, T. Gudmundsen, D. Hover, J. Miloshi, R. Slattery, F. Yan, J. Yoder, T. P. Orlando, S. Gustavsson, and W. D. Oliver. Thermal

and residual excited-state population in a 3d transmon qubit. *Phys. Rev. Lett.*, 114:240501, Jun 2015.

- [183] Maxime Boissonneault, J. M. Gambetta, and Alexandre Blais. Nonlinear dispersive regime of cavity qed: The dressed dephasing model. *Phys. Rev. A*, 77:060305, Jun 2008.
- [184] Erik Lucero, M. Hofheinz, M. Ansmann, Radoslaw C. Bialczak, N. Katz, Matthew Neeley, A. D. O'Connell, H. Wang, A. N. Cleland, and John M. Martinis. Highfidelity gates in a single josephson qubit. *Phys. Rev. Lett.*, 100:247001, Jun 2008.
- [185] L. S. Onyshkevych, W. F. Kosonocky, and A. W. Lo. Parametric Phase-Locked OscillatorCharacteristics and Applications to Digital Systems. *IRE Transactions* on Electronic Computers 0367-9950, EC-8(3):277–286, September 1959.
- [186] Z. Wang, S. Shankar, Z.K. Minev, P. Campagne-Ibarcq, A. Narla, and M.H. Devoret. Cavity attenuators for superconducting qubits. *Phys. Rev. Applied*, 11: 014031, Jan 2019.
- [187] E. Purcell. Spontaneous emission probabilities at radio frequencies. Physical Review, 69(11-12):674–674, June 1946.
- [188] A. Kringhøj, L. Casparis, M. Hell, T. W. Larsen, F. Kuemmeth, M. Leijnse, K. Flensberg, P. Krogstrup, J. Nygård, K. D. Petersson, and C. M. Marcus. Anharmonicity of a superconducting qubit with a few-mode josephson junction. *Phys. Rev. B*, 97:060508, Feb 2018.
- [189] F. E. Terman. Radio engineers handbook. McGraw-Hill, New york, 1945.
- [190] Eyob A. Sete, John M. Martinis, and Alexander N. Korotkov. Quantum theory of a bandpass purcell filter for qubit readout. *Phys. Rev. A*, 92:012325, Jul 2015.
- [191] Arjan F. van Loo, Arkady Fedorov, Kevin Lalumière, Barry C. Sanders, Alexandre Blais, and Andreas Wallraff. Photon-Mediated Interactions Between Distant Artificial Atoms. *Science0036-8075*, 1095-9203, 342(6165):1494–1496, December 2013.
- [192] S. Diehl, A. Micheli, A. Kantian, B. Kraus, H. P. Büchler, and P. Zoller. Quantum states and phases in driven open quantum systems with cold atoms. *Nature Physics1745-2481*, 4(11):878–883, November 2008.

- [193] Jaeyoon Cho, Sougato Bose, and M. S. Kim. Optical pumping into many-body entanglement. *Phys. Rev. Lett.*, 106:020504, Jan 2011.
- [194] Shwetank Kumar and David P. DiVincenzo. Exploiting Kerr cross nonlinearity in circuit quantum electrodynamics for nondemolition measurements. *Physical Review B*, 82(1):014512, July 2010.
- [195] Lukas Neumeier, Martin Leib, and Michael J. Hartmann. Single-photon transistor in circuit quantum electrodynamics. *Phys. Rev. Lett.*, 111:063601, Aug 2013.
- [196] Marco T. Manzoni, Florentin Reiter, Jacob M. Taylor, and Anders S. Sørensen. Single-photon transistor based on superconducting systems. *Phys. Rev. B*, 89: 180502, May 2014.
- [197] Jie-Qiao Liao, Jin-Feng Huang, Yu-xi Liu, Le-Man Kuang, and C. P. Sun. Quantum switch for single-photon transport in a coupled superconducting transmissionline-resonator array. *Phys. Rev. A*, 80:014301, Jul 2009.
- [198] Christian Kraglund Andersen and Klaus Mølmer. Circuit qed flip-flop memory with all-microwave switching. *Phys. Rev. Applied*, 3:024002, Feb 2015.