

Master Thesis

Fast Flux Control Of A Transmon Qubit In A Three-Dimensional Cavity Architecture

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Abstract

Fast flux control on a SQUID based transmon qubit has been a hard task in threedimensional cavity architectures. The most simple option of putting a coil outside a copper cavity suffers from counteracting eddy currents in the cavity walls. Eddy currents prevent fast magnetic field changes from penetrating the conducting cavity and thus controlling the qubit inside becomes impossible. Another option is provided by flux bias lines, which are U-shaped wires put only ten to one hundred micrometers next to a qubit inside the cavity. Due to the close distance the flux bias line is capacitively coupled to the qubit and offers an additional channel for decay. The enhanced decay of the qubit is partially prevented by complex filtering, however filtering is never perfect. This thesis introduces a new approach for fast flux control on a transmon qubit in a three-dimensional cavity architecture. A magnetic hose is used to guide a magnetic pulse from the outside to the inside of a three-dimensional microwave cavity. The magnetic pulse is generated by a coil outside the cavity and sent through the hose to control the transition frequency of a SQUID based transmon qubit inside the cavity on a timescale of hundreds of nanoseconds.

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Introduction

The original task for this thesis was to build fast tunable coils. These coils should be able to generate a magnetic field of maximally some nanotesla at a point of interest. The magnetic field at the point of interest should be switched fast to arbitrary values within the maximum range. Fast switching between two arbitrary field strengths has to be below some hundreds of nanoseconds. First estimations show that magnetic fields of some nanotesla are easy to reach and the switching below hundreds of nanoseconds is feasible either. So far so good, building such fast tunable coils is possible. However, there is a tricky part concerning the point of interest, where the field should be applied.

The point of interest is enclosed by a highly conducting box, a microwave cavity made out of copper. This constraint leads to problems as soon as a magnetic field is switched fast between two values. Whenever the magnetic flux through a conductive surface changes, eddy currents appear in the surface and counteract the change in magnetic flux. The counteracting effect of eddy currents increases proportional to the change of flux in time. As a consequence the conducting cavity walls act as a low pass filter for the applied magnetic field and switching below hundreds of nanoseconds becomes impossible. Full control of a magnetic field inside a microwave cavity will be unreachable, if the magnetic source is placed outside the cavity.

But, why does someone want to have full control of a magnetic field inside a conductive box? The answer is, to control a superconducting qubit. At the point of interest, the centre of a microwave cavity, a superconducting qubit is placed. The qubit is sensitive to magnetic flux and thus its transition frequency is tunable by an external applied magnetic field. Full control of the magnetic field inside the cavity results in full control on the qubit's transition frequency. The magnetic field has to be applied from the outside of the microwave cavity, since a coil inside the cavity couples capacitively to the qubit and the microwave field. The coupling enhances the decay of the qubit and offers an additional channel for microwave losses. Controlling a qubit's transition frequency is a necessary ingredient for the realisation of quantum computation and simulation. This thesis motivates the task of full flux control on a superconducting qubit inside a microwave cavity, analyses the issues that come along and gives a solution.

In the first chapter the qubit is introduced and motivated by the idea of quantum computation and simulation. It is shown, how a qubit can be realised experimentally in a cavity quantum electrodynamic system. The relevant mathematical background is derived, leading to the Jaynes-Cummings Hamiltonian in the dispersive limit. The dispersive limit is used to read out the state of the qubit. Instead of using a cavity quantum electrodynamic system one can use a circuit quantum electrodynamic system to realise and control a qubit. The mathematical background is the same in both systems, despite the totally different experimental setup. In a cavity electrodynamic system atoms are interacting with light in an optical cavity, in a circuit electrodynamic system superconducting quantum circuits are interacting with microwaves in a microwave cavity. Superconducting quantum circuits can be designed nearly arbitrarily and therefore offer a large tunability in their parameters. Such kind of circuits can be used to built artificial atoms with a huge dipole moment. Therefore superconducting quantum circuits can be used as qubits and are a promising candidate for realising quantum computation and simulation. The working principle of the transmon, a special superconducting qubit, is explained in the first chapter.

The second chapter explains how to guide a magnetic field from the outside to the inside of a highly conducting cavity. First, the generation of magnetic fields is explained in the static case by the law of Biot-Savart. Switching on a static magnetic field is discussed next and first experimental limits are introduced. Everything seems fine until eddy currents are considered. A theoretical model on eddy currents is introduced and the effect is measured. Two widely known solutions to compensate the effect of eddy currents are presented. However both cannot be implemented in this setup. A way to circumvent all theses issues is offered by a magnetic hose. A magnetic hose is a device that transfers an arbitrary magnetic field between two points, that are connected by the hose. The functionality of the hose is investigated systematically, leading to promising results.

The final results of fast flux control of a superconducting qubit are shown in the third chapter. In the first section the experimental setup is explained for different measurements and it is shown, how a qubit is measured and characterised. The following section discusses the generation of fast magnetic pulses. The measurement scheme for investigating the fast flux control of a superconducting qubit is introduced. Finally the results are presented, indicating a tunability below two hundred nanoseconds.

Chapter 1

From Cavity To Circuit Quantum Electrodynamics

1.1 Overview

This chapter presents the required theory to understand the flux control of a superconducting quantum bit. Therefore the quantum bit is introduced first and the idea to use it for computation and simulation is motivated. Within this section, some quantum gates operating on the quantum bit are explained. These gates are necessary to perform the measurements shown in chapter 3.

The next section introduces the field of cavity quantum electrodynamics by considering the coupling between a single atom and a single electromagnetic field mode. In case the atom is driven between two of its states only, it is treated as spin-1/2 particle. This assumption and some additional approximations lead from a general linearised Hamiltonian to the Jaynes-Cummings model. There the cavity photons and the atom form a joint state, which obscures their individual character. However, in the dispersive limit of the Jaynes-Cummings model their individual character emerges. As a consequence the photon number or atom state in the cavity can be measured individually by performing a so called dispersive read-out. Actually, the dispersive read-out scheme can be applied to any physical system, where a spin-1/2 particle is coupled to a harmonic oscillator.

Circuit quantum electrodynamics provides such a system and is introduced in the following section. Instead of atoms, circuit quantum electrodynamics uses superconducting quantum circuits as artificial atoms and couples them to a single microwave mode in a microwave resonator. The working principle of any superconducting quantum circuit is based on the Josephson effect caused by a Josephson junction.

A superconducting quantum bit is basically an anharmonic oscillator and is explained by comparing it to a harmonic oscillator in the language of electronic circuits. The harmonic oscillator of choice is the LC-circuit, which is realised by a microwave cavity. Subsequently the relevant properties of such a cavity are discussed. It is quantised by introducing flux and charge operators.

Next the transmon, a special superconducting quantum bit, is introduced and

quantised. It consist of a single Josephson junction and has a fixed transition frequency. To make this transition frequency tunable, one adds a second Josephson junction in parallel to form a loop. The transmon transition frequency is then sensitive to the flux through the loop and becomes tunable.

Finally the microwave cavity and the transmon are coupled to form a circuit quantum electrodynamic system. The system is described by the same mathematics as in the case of an atom coupled to a single electromagnetic field mode. Therefore it is possible to detect the state of the transmon by a dispersive read-out and use the system for quantum simulation and computation.

1.2 Quantum Computation And Simulation

Nowadays computers are based on classical mechanics using classical bits for information processing. Since quantum mechanics is a more general theory including classical mechanics, one can make us of additional properties of quantum mechanics to provide more power for computation. Entanglement between quantum systems and the superposition principle make parallel processing possible. In theory quantum algorithms outperform some of the today known algorithms for classical computation in tasks like searching, factorising or optimising [1]. Basic principals have been verified experimentally, but there is still a lot of development and improvement necessary to beat classical computation.

To realise a universal quantum computer one has to fulfil the first five criteria stated by Di Vincenzo [2]. Therefore one requires:

- 1. a scalable physical system with well characterised qubits
- 2. the ability to initialize the state of the qubits to a simple fiducial state
- 3. long relevant decoherence times, much longer than the gate operation time
- 4. a "universal" set of quantum gates
- 5. a qubit-specific measurement capability

In principle these criteria are fulfilled in ion trap and superconducting circuit experiments. Still both could not manage to sufficiently control large systems, that outperform a classical computer.

Besides the hot topic of building a universal quantum computer, specific binary quantum systems can be used for simulating other quantum systems. Quantum simulations on classical computers need a huge amount of resources. Even relatively small systems consisting for example out of fifty particles will become impossible to simulate on a classical supercomputer if an interaction between all particles is considered. A system of *n* spin-1/2 particles can take on a superposition of 2^n states, each having a different complex amplitude. Therefore space in the order of 2^n classical bits is needed to save the state of *n* quantum bits, even more space and time is then needed to perform calculations on this huge number of bits.

Richard Feynman was the first one who pointed out that it might be more efficient to simulate quantum systems with other well controlled quantum systems [3]. In case of a specific quantum simulation, it is not required to build a universal quantum computer. The system of interest is actually built for a specific class of problems only. Using such a system may be more efficient than using classical supercomputers for solving the same problem.

The experimental development of quantum simulators is promising and they might outperform classical computation in the next ten years [4]. Superconducting qubits are one way to realise quantum computation and simulation. The theory of their physics is investigated in the following sections, starting with introducing the closely related topic of cavity quantum electrodynamics. In this section the idea of information processing is introduced first. Then the concept of the quantum bit and gates is discussed shortly. Finally the necessary gate operations for this thesis are explained.

1.2.1 The Idea Of Computation

Computation is processing of information. Information is always connected to a physical system and it is gained from distinguishable states of that system. The smallest system that acts as information carrier is a system with two distinguishable states. To do computation on such a system one introduces the binary digit, shortly called bit. It is is the unit of information and can take on the values 0 or 1 for convenience.

Any information of a two state system is represented by a single bit. Therefore a two state system can be encoded into a single bit. Of course one can think of a larger system that takes on more than two states and use it for computation. The information of such a system can always be encoded into a sequence of bits. A sequence of *n* bits has 2^n different ways to be ordered, so 2^n different states can be encoded. The ordering of a sequence is spatial or temporal.

The process of computation is done by an algorithm. An algorithm is a specific ordering of operations, that act on a given bit sequence. Therefore the algorithm fulfils a predetermined task on an input sequence and yields an output sequence. From decoding the output sequence new information is gained.

To do computation one needs two state systems acting as bits that can be initialised and keep their assigned state sufficiently long. Next operations acting on the bits are required to realise an algorithm. Finally a measurement process has to be available to detect the output bit sequence.

1.2.2 The Quantum Bit

Quantum computation is processing of information gained from quantum systems. There the unit of information is the quantum bit, abbreviated qubit. A qubit distinguishes between two quantum states of a quantum system. The two distinguishable states are written as $|0\rangle$ and $|1\rangle$ in analogy to classical computation. In contrary to classical systems, quantum mechanics allows superposition of two quantum states to form a new state. Therefore a quantum bit can be set to an arbitrary state of the form

$$|\Psi\rangle = \alpha |0\rangle + \beta |1\rangle \quad , \tag{1.1}$$

where α and β are complex numbers that satisfy $|\alpha|^2 + |\beta|^2 = 1$ for normalisation. The state $|\Psi\rangle$ is a vector in a two dimensional Hilbert space \mathscr{H} , where $|0\rangle$ and $|1\rangle$ form an orthonormal basis.

A useful representation of a single qubit state is

$$|\Psi\rangle = \cos(\theta/2)|0\rangle + e^{i\varphi}\sin(\theta/2)|1\rangle \quad . \tag{1.2}$$

In this representation the state is visualised on a sphere, the so called Bloch sphere. It is illustrated in figure 1.1. The angles $\theta \in [0, \pi]$ and $\varphi \in [0, 2\pi)$ are the same as in



Figure 1.1: Bloch sphere. The Bloch sphere is used to illustrate a single qubit state $|\Psi\rangle = \cos(\theta/2)|0\rangle + e^{i\varphi}\sin(\theta/2)|1\rangle$.

spherical coordinates. Every point on the Bloch sphere represents a single qubit state. The coherent superposition of the states $|0\rangle$ and $|1\rangle$ is given by θ and the relative complex phase between them is given by φ .

Multiple qubits are described in a joint Hilbert space $\mathcal{H} = \bigotimes_{i=1}^{n} \mathcal{H}_{i}$ by the tensor product of every single qubit Hilbert space \mathcal{H}_{i} . The resulting state

$$|\Psi\rangle = \sum_{i=1}^{n} c_i |i\rangle \tag{1.3}$$

is a 2*n* dimensional vector, where the condition $\sum_{i=1}^{n} |c_i|^2 = 1$ normalises the coefficients. The *i* in each basis state $|i\rangle$ is sometimes written in binary notation to see the connection to each qubit.

One may think that infinite information can be encoded into a single qubit, because there are infinite possibilities to choose the complex numbers α and β . This is only partially true. Even if the information can be encoded arbitrarily, the decoding does not work that easy. The state of a quantum system can only be determined after performing a measurement on the system.

A measurement is related to a specific basis. Such a basis consists of two orthonormal states in case of a single qubit, like $|0\rangle$ and $|1\rangle$ form the σ_z -basis along the *z*-axis. The measurement projects the initial qubit state into one of the two orthonormal basis states. This happens with the probability given by the square of the absolute value of the corresponding complex amplitudes α and β .

A single shot measurement result is one of two eigenvalues connected to the basis. The corresponding eigenvalues appear with probability $|\alpha|^2$ or $|\beta|^2$. Therefore only the states $|0\rangle$ or $|1\rangle$ can be concluded from the obtained eigenvalue after a single shot measurement in the σ_z -basis. The amplitudes α and β are obtained to arbitrary precision either by measuring a sufficiently large ensemble of qubits prepared in the

same state or by repeatedly measuring a single qubit, that is prepared in the same state again after the measurement.

1.2.3 Single Qubit Gates

A single qubit can take on one state out of infinite possible states, which are represented on the Bloch sphere. To transfer a state from one point on the sphere to another one, infinite ways are possible. Consequently there exists an infinite number of single qubit operations to control the state. In the language of quantum computation, an operation that transfer the quantum state is called gate.

Mathematically single qubit gates are represented by unitary 2×2 matrices acting on the two-dimensional single qubit state. Any arbitrary complex 2×2 matrix can be represented by a linear combination of the identity and the Pauli matrices:

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$
(1.4)

Therefore the operation on a single qubit is represented by the operator

$$H = \frac{1}{2}h_0\mathbb{1} + \frac{1}{2}(h_x\sigma_x + h_y\sigma_y + h_z\sigma_z) = \frac{1}{2}\vec{h}\vec{\sigma}$$
(1.5)

in the Pauli basis.

The coefficient h_0 is neglected, because it leads to a constant energy shift for any qubit state and thus it is physically irrelevant. As a result the arbitrary single qubit operation is the scalar product between the normalised Pauli vector $\vec{\sigma}/2$ and a real three-dimensional vector \vec{h} . One can rewrite \vec{h} by a normal vector \vec{n} multiplied by the normalisation Ω to get

$$H = \frac{1}{2}\Omega \vec{n}\vec{\sigma} \tag{1.6}$$

in the static case.

The arbitrary single qubit gate results from solving the time dependent Schrödinger equation considering the static operator (1.6). The resulting unitary

$$U(t) = e^{-i\frac{H}{\hbar}t} \quad \Leftrightarrow \quad R_{\vec{n}}(\Omega t) = \cos\left(\frac{\Omega t}{2}\right)\mathbb{1} - i\sin\left(\frac{\Omega t}{2}\right)\vec{n}\vec{\sigma} \tag{1.7}$$

is equivalent to a rotation $R_{\vec{n}}(\Omega t)$ on the Bloch sphere¹. The vector \vec{n} defines the rotation axis and Ωt the amount of rotation.

In the experiments in chapter 3 only two rotations are necessary to first characterise the qubit and to investigate the qubit's behaviour on a fast flux pulse afterwards. The required rotations are performed in the xz-plane about the y-axis on the Bloch sphere. As a consequence the rotation matrix

$$R_{y}(\Omega t) = \begin{pmatrix} \cos\left(\frac{\Omega t}{2}\right) & -\sin\left(\frac{\Omega t}{2}\right) \\ \sin\left(\frac{\Omega t}{2}\right) & \cos\left(\frac{\Omega t}{2}\right) \end{pmatrix}$$
(1.8)

¹The equivalence is shown by splitting the exponential function's Taylor series into even and odd terms. Since the square of any Pauli matrix $\sigma_i \sigma_j = \delta_{ij} \mathbb{1} + i \sum_{k=1}^3 \varepsilon_{ijk} \sigma_k$, it follows that $(\vec{n}\vec{\sigma})^2 = \mathbb{1}$, where δ_{ij} is the Kronecker delta and ε_{ijk} is the Levi-Civita symbol.

is simplified. If the argument $\Omega t = \pi$, the rotation matrix $R_y(\pi) = \sigma_x$ and corresponds to a spin flip in the σ_z -basis. This operation is called π -pulse. It is used for instance to excite the qubit from the ground state $|0\rangle$ to the excited state $|1\rangle$. If the argument $\Omega t = \pi/2$, the rotation matrix $R_y(\pi/2) = \frac{1}{\sqrt{2}}(1 - i\sigma_y)$. This operation is called $\pi/2$ -pulse. It is used for instance to bring the qubit from the ground state $|0\rangle$ to the superposition state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$.

In case of more than one qubit, multiple qubit gates are needed to control the interaction among each qubit. Multiple qubit gates are described by unitary $2^n \times 2^n$ matrices, where *n* is the number of related qubits. Again there are infinite possible multi qubit gates, like there are infinite possible single qubit gates. Besides the infinite number of possible qubit gates, specific algorithms only need a few relevant gates. There even exist sets of gates that are universal, meaning that any qubit gates. An experimental realisation of a universal set of quantum gates is the key to perform flexible quantum simulations or to even build a universal quantum computer one day.



Figure 1.2: Sketch of a basic cavity QED system. Two mirrors (blue) form the cavity, comprising a single cavity mode (red). The atom (white) inside the cavity is treated as a two level system with a ground state $|g\rangle$ and an exited state $|e\rangle$. The cavity mode and the atom are coupled and interact depending on the interaction rate or coupling strength *g*. Losses appear in two ways. On the one hand cavity photons may leak through the cavity with a rate κ and on the other hand the atom decays with a rate γ .

1.3 Cavity QED

The interaction between matter and light is described by the theory of quantum electrodynamics (QED). This section gives a glimpse into a special but very important case: the interaction between a single atom treated as a two level system and a single electromagnetic field mode trapped inside a cavity. Enclosing a single atom and photons in a cavity decouples them from the noisy outside world. Therefore it is possible to have a very precise control on the quantum behaviour, making quantum information processing feasible. The research field is called cavity QED due to the system's setup, depicted in figure 1.2. Cavity QED has opened a wide range of new research fields that are still growing [5].

First theoretical analysis of cavity QED was done by Purcell [6]. He assumed a single atom in a cavity with highly conductive walls. From this model he predicted the Purcell effect, which states that the spontaneous emission rate of an atom is enhanced by putting it into a resonant cavity. Soon more theoretical works to this topic followed. One of them was published by Casimir, who calculated the force between two conductive plates in free space [7]. The result is an attracting force between the plates, known as Casimir force. The cause of this force is a higher mode density because of vacuum fluctuations exterior the plates. Therefore a cavity isolates an atom inside from the high mode density and noise in free space. An important remark to the Purcell effect was published by Kleppner [8], where he predicted that spontaneous decay of an atom inside a cavity can also be inhibited. This is the case when the cavity is strongly off resonant with the atom's transition frequency and the mode density resonant with the atom is reduced.

All these proposed models lead to experiments where single Rydberg atoms interact with single photons in a microwave cavity [9]. Since then the term cavity QED has been used to describe physical systems like that. These systems made experiments possible, where photons enclosed in a cavity are counted without destroying them. From that, precise photon state preparation, control on single atoms and entanglement between atoms and photons has been developed [5]. Soon these experiments were achieved for atoms in the optical regime [10] and also for superconducting circuits [11].

1.3.1 Hamilton Formalism Of Cavity QED

Cavity QED investigates the interaction between a single atom and a single mode field in a cavity. Such a system is described by the Hamiltonian

$$H_{ac} = \hbar\omega_c a^{\dagger} a + \frac{\hbar\omega_a}{2}\sigma_z + \hbar g(\sigma_+ + \sigma_-)(a + a^{\dagger})$$
(1.9)

in the linear regime. A clear derivation of this Hamiltonian is given in [12] or [13].

The first term models the cavity's field mode by a quantised harmonic oscillator with frequency ω_c , where a^{\dagger} and a are the creation and annihilation operators, respectively. The field modes inside a cavity are quantised spatially by the cavity's geometry. Therefore the cavity acts as a filter. Only single modes from an external light source can enter with high probability feeding the cavity with photons.

The second term describes the atom as two-level system or qubit in the language of a spin-1/2 particle. This approximation is valid, as the atom is driven on a single transition between two of its states only. The two states are usually referred to as $|g\rangle$ and $|e\rangle$ for the ground and excited state, respectively. The transition frequency ω_a is connected to the energy difference between the two states via \hbar , the Planck constant in units of 2π . Here the Pauli matrix $\sigma_z = |e\rangle \langle e| - |g\rangle \langle g|$ describes the state dependent energy shift.

The third term characterises the interaction between the atom and single photons of the cavity mode. The coupling strength *g* determines the strength of the interaction. The value of *g* is proportional to the field strength inside the cavity and the atoms dipole moment. Higher order terms of the multipole expansion are neglected, since the dipole moment between the atomic transition is by far the leading term. The operators $\sigma_+ = |e\rangle \langle g|$ and $\sigma_- = |g\rangle \langle e|$ describe transitions between the atomic states.

One thing to keep in mind is that the Hamiltonian (1.9) describes a perfect systems without any losses. In a real experiment, however, losses arise. Cavity photons are leaking out of the cavity at a rate κ and the atom decays at a rate γ . These losses lead to decoherence and are the main part why it is difficult to control quantum systems. Here the loss rates will be considered small with respect to the coupling strength and thus are neglected.

In general the Hamiltonian (1.9) models the linear interaction between a harmonic oscillator and a spin-1/2 particle. Therefore it is used to describe many other physical systems like ions [14], quantum dots [15], nanomechanical systems [16] and superconducting circuits [17].

1.3.2 The Jaynes-Cummings Model

A special case of the Hamiltonian (1.9) is achieved when the detuning $\Delta = \omega_c - \omega_a$ between cavity frequency and atom transition frequency is close to zero. The terms $\sigma_+ a^{\dagger}$ and $a\sigma_-$ are then very unlikely to happen. These terms become highly off-resonant transitions in the interaction picture and can be neglected due to energy conservation in the rotating wave approximation. The result is the Jaynes-Cummings Hamiltonian

$$H_{JC} = \hbar \omega_c a^{\dagger} a + \frac{\hbar \omega_a}{2} \sigma_z + \hbar g (\sigma_+ a + a^{\dagger} \sigma_-) \quad . \tag{1.10}$$

In this regime, where $\Delta \approx 0$, the cavity is resonant with the atom transition frequency. Cavity photons and the atom form a joint state out of a family of states, that follow from diagonalising the Jaynes-Cummings Hamiltonian (1.10). The resulting eigenstates and eigenvalues are the so called dressed states and dressed eigenvalues

$$|\pm,n\rangle = \frac{1}{\sqrt{2}}(|g,n+1\rangle \pm |e,n\rangle) \tag{1.11}$$

$$E_{\pm,n} = \hbar\omega_c \left(n + \frac{1}{2}\right) \pm 2\hbar g \sqrt{n+1} \tag{1.12}$$

for $\Delta = 0$.

The system is in the so called *resonant regime* for $\Delta \ll g$. If the coupling strength g is sufficiently large, meaning larger than the loss rates κ and γ , the cavity photons and the atom can freely exchange energy. In the resonant regime the atom may absorb a photon and emit it back to the cavity, performing vacuum Rabi oscillations. Therefore g is a measure for the interaction rate between the atom and the cavity photons before losses prevail.

1.3.3 Dispersive Read-Out

In the case of $\Delta \gg g$ the cavity QED system is in the *dispersive regime*, where the cavity state and the atom state show only weak properties of a dressed state and thus are considered separately. Due to the strong detuning and the relatively small coupling strength, the atom does actually not absorb or emit a cavity photon. The Jaynes-Cummings Hamiltonian is approximated by the dispersive Hamiltonian

$$H_{disp} = \hbar\omega_c a^{\dagger}a + \frac{\hbar\omega_a}{2}\sigma_z + \hbar\frac{g^2}{\Delta}\left(a^{\dagger}a + \frac{1}{2}\right)\sigma_z \tag{1.13}$$

in this regime.

The approximation is found by first applying a unitary transformation $U = e^{\eta}$ to the Jaynes-Cummings Hamiltonian. Then the Baker-Campbell-Hausdorff formula [18] is used to expand the transformed Hamiltonian to second order. After that, η is chosen such that first order terms in the expansion drop out. First order terms have to drop out, because they do not add any energy shift. This can be easily seen by applying time independent first order perturbation theory to the interaction term.

Equation (1.13) is simplified and rewritten to

$$H_{disp} = \hbar \omega_c a^{\dagger} a + \hbar \tilde{\omega}_a \sigma_z + \hbar \chi \sigma_z a^{\dagger} a \tag{1.14}$$



Figure 1.3: Dispersive read-out scheme. In a) the transmission of the pure cavity is sketched. Adding a two-level atom to the cavity shifts the transmission peak in the dispersive limit depending on the qubit's state as sketched in b). The red peak presents the transmission when the qubit is in the ground state and the blue peak is referred to the excited state. For the dispersive read-out the transmission at $\omega_c + \chi$ is measured. According to the purple dots the transmission decreases, if the qubit is in the excited state following the purple arrow.

by introducing the dispersive frequency shift $\chi = \frac{g^2}{\Delta}$ and the effective atom transition frequency $\tilde{\omega}_a = (\omega_a + \chi)/2$. The third term again considers the interaction between atom and cavity. Now however, it is called dispersive energy shift and depends on the atom's state and the photon number in the cavity.

The dispersive regime allows one to perform measurements on the atom's state or the cavity photon number by preserving the measured state. This kind of measurement is called quantum non demolition (QND) measurement. If the interest is in the atom's state, the cavity's transmission at a particular frequency has to be measured. The dispersive Hamiltonian

$$H_{disp} = \hbar (\omega_c + \chi \sigma_z) a^{\dagger} a + \hbar \tilde{\omega}_a \sigma_z$$
(1.15)

shows in this form a frequency shift for the cavity depending on the atom's state, see figure 1.3. To access a high measurement resolution, $\chi \approx \kappa$ and $\gamma \ll \kappa$, χ has to be valid.

The dispersive Hamiltonian (1.15) is essential for quantum computation. It provides a read-out scheme for the atom or qubit. As said before, starting from equation (1.9) all derivations done so far are valid for all kind of systems where a harmonic oscillator interacts with a spin-1/2 particle. Therefore it can be applied to a superconducting qubits coupled to a coherent microwave field mode as explained in the next section.

$$|\Psi_1\rangle = \sqrt{n_1} e^{i\phi_1} \qquad |\Psi_2\rangle = \sqrt{n_2} e^{i\phi_2}$$

Figure 1.4: Sketch of a Josephson junction. The grey areas symbolise superconducting islands that are separated by an insulator represented as blue area. In each superconducting island Cooper pairs are described by a joint quantum state $|\Psi_1\rangle$ and $|\Psi_2\rangle$ respectively.

1.4 Circuit QED

Another way to realise a physical setup like in cavity QED is offered by circuit QED. In circuit QED superconducting quantum circuits are interacting with the single modes of a microwave resonator. Superconducting quantum circuits can be designed in different ways to perform various tasks. They act as resonators, nanomechanical systems [19] or qubits [17]. Here the focus is on the interaction between a single superconducting qubit and photons in a microwave cavity at some GHz.

The essential element of superconducting quantum circuits to get into the quantum regime is the Josephson junction. A Josephson junction is a nanoscale insulator between two superconducting islands, illustrated in figure 1.4. In a superconducting island all Cooper pairs form a joint quantum state

$$|\Psi\rangle = \sqrt{n}e^{i\phi} \tag{1.16}$$

where *n* is the density of Cooper pairs and $\tilde{\phi}$ the wavefunction's phase. Whenever a Cooper pair tunnels from one island through the junction to the other, the state in each island is changed.

This was first theoretically predicted by Brian D. Josephson and is now known as the Josephson effect [20]. The Josephson effect is summarised in two equations

$$I = I_c \sin(\phi) \tag{1.17}$$

$$\frac{\partial \phi}{\partial t} = \frac{2\pi}{\Phi_0} U \tag{1.18}$$

which are referred to as the first and the second Josephson equation respectively. Here *I* and *U* are the current and voltage across the junction, I_c is the junction's critical current and Φ_0 is the flux quantum. The current and voltage are connected via the relative phase ϕ between the two wavefunctions in each superconductor.

The importance of the Josephson junction comes from the fact, that it is the only known non-dissipative and non-dephasing element for circuits that provides non-linearity [21]. This non-linearity is essential to define a qubit, whereas the other properties guarantee its stability.

A Cooper pair may tunnel through the junction because of some external excitation. Therefore a charge difference between the islands and a phase difference between the wavefunction in each island appear. The charge difference causes a capacitive energy going linearly with the charge difference. The phase difference



Figure 1.5: Potential energy and circuit representation of an LC-circuit in a) and a transmon in b). The LC-circuit on the left side behaves like a harmonic oscillator with a quadratic potential. Therefore the quantised energy steps are separated by $\hbar\omega_r$ equidistantly. The capacitively shunted Josephson junction on the right side has two equivalent circuit representations. The cross symbolises the non-linear inductance and the cross in the box represents the total junction including the capacitance. The potential energy of a transmon is periodic and has a cosine shape. Therefore the quantised energy steps are not equidistant. A transmon can be used to define a two-level system with a transition frequency ω_q between its lowest states.

causes an inductive energy that is proportional to the cosine of the phase difference. Those properties are similar to an LC-circuit, where also a capacitive and an inductive part are combined. However, there is an essential difference.

The capacitor and inductance in an LC-circuit are linear elements. Thus it behaves as a harmonic oscillator at a resonance frequency $\omega_r = \frac{1}{\sqrt{L_r C_r}}$ given by the capacitance C_r and inductance L_r of the circuit. A Josephson junction is described by a linear capacitance C_J and a non-linear inductance L_J . Because of the non-linear inductance the junction is used to built an anharmonic oscillator that can be used as two-level system, qubit or artificial atom. Adding a shunt capacitance C_s to the junction fixes charge dispersion and thus stabilises the anharmonic oscillator. This configuration is known as transmon [22]. Figure 1.5 shows the circuit representation of both, an LC-circuit and a transmon.

Circuit QED investigates the interaction between superconducting circuits. In the following, two circuits as depicted in figure 1.5 are investigated separately and connected afterwards. First, the three-dimensional microwave cavity is presented as a harmonic oscillator and mathematically described as an LC-circuit. Second, the transmon qubit in general and its flux sensitive version is introduced. Finally, the transmon is coupled to a microwave cavity and their interaction is shown to be the same as in the cavity QED system discussed before.

1.4.1 Microwave Cavities

In general a microwave cavity can be designed in various forms. For example there are transmission line, coplanar waveguide or three-dimensional microwave cavities. All cavities have the same mathematical description and can be modelled by LC-circuits with a resonance frequency in the GHz regime. The cavity's task is to provide an environment that shields a superconducting qubit from noise and to enable state control and state read-out on the qubit. This section focuses on the properties of a three-dimensional cavity architecture.

Geometry Aspects

A three-dimensional cavity has an inner volume V = abd. It forms a rectangular microwave resonator, where *a*, *b* and *d* are its dimensions. Microwaves are sent into the cavity for qubit excitation and read-out. Depending on the cavity's inner volume only particular frequencies

$$f_{mnl} = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{l}{d}\right)^2}$$
(1.19)

are able to enter the cavity without being heavily attenuated [23]. Here c is the speed of light in vacuum. The integers m, n and l number the anti-nodes of the standing electric field inside the cavity along the x-, y- and z-axis respectively.

Usually one of the cavity's fundamental mode is used for read-out. A fundamental mode has one of the integers m, n or l set to zero, the other two are set to one in case of a rectangular resonator. The cavity is designed such, that the lowest fundamental mode's frequency $f_r = 2\pi\omega_r$ is in the range of some GHz and higher modes are far away from that frequency.

One of the inner cavity dimensions can be set close to zero, because the field is constant along the third. Those two dimensions control the frequency according to equation (1.19). The negligible dimension has to be the dimension that points along the qubit's dipole axis. In other words the cavity's inner surface that is perpendicular to the qubit's dipole axis has to be the largest of the rectangular resonator. If this is not the case, the fundamental mode's electric field and the qubit's dipole moment are orthogonal. The discussed aspects are represented in figure 1.6.

In principal the two dimensions of the relevant surface are allowed to have different lengths. One has to consider that if one dimension is smaller than $\frac{c}{2f_r}$, the fundamental frequency is pushed to high. In this case the other dimension cannot be large enough to compensate the short length of the first dimension. An imbalance in the dimensions has one disadvantage. Higher modes appear at frequencies close to the fundamental mode. This should be avoided, because these unwanted frequencies could interact additionally with the qubit and lead to decoherence or excite it to higher states [24]. The best way to push the frequencies of higher modes up is to use similar dimensions for both sides.



Figure 1.6: Electric field in a rectangular waveguide cavity. A cavity with dimensions a, b and d is represented in a). The qubit is fabricated on a chip (blue). It couples to the TE₁₁₀ mode (red). In b) the electric field distribution of the TE₁₁₀ mode is illustrated, resulting from a numerical calculation. The field amplitude is maximal (red) in the center of the cavity. It decreases cosine-shaped leaving the centre and vanishes at the cavity walls.

Manufacturing Aspects

The field in a microwave cavity has an electric and a magnetic component. Whenever the electric component is at its maximum, the magnetic component is zero and vice versa. The field inside the cavity induces currents in the cavity walls. These currents oscillate at the same frequency as the microwave field, keeping the field inside alive. As soon as the currents dissipate the mode vanishes. Therefore material of high conductivity like OFHC copper or superconductors like ultra pure aluminium is used to keep the currents oscillating in the cavity walls for long times.

A cavity of high internal quality factor is required to prevent the currents from dissipating. The internal quality factor

$$Q_i = \frac{P_{tot}}{P_{dis}} \tag{1.20}$$

is a measure for the ratio of total power that is put into the cavity and the dissipated power from the cavity. Cavities made out of ultra pure aluminium reach inner quality factors above 10⁶, where cavities made out of OFHC copper have internal quality factors in the order of 10⁴. OFHC copper cavities that are electroplated with indium have a higher internal quality factor than the ultra pure aluminium cavities [25] in the same order of magnitude. The best quality factors are achieved in special cylindrical resonators [26]. These resonators do not have any single seam in the cavity walls, which is the main reason for dissipation in a three dimensional resonator.

Rectangular waveguide resonators are manufactured having at least one seam. The usual process is to cut one solid block of metal into two pieces and mill a half cavity into each of them. Both milled blocks are then put together to form the actual microwave cavity. Therefore at least one seam is unavoidable to get a cavity inside a solid block of metal. The seam causes dissipation, but when placing it right and enclosing it with indium the dissipation becomes negligible.



Figure 1.7: In a) the current density \vec{J} (yellow) oscillates on the inner cavity walls and keeps the electromagnetic field, \vec{E} (red) and \vec{B} (blue), inside the cavity alive. Cutting the cavity along the *x*-*y*-plane suppresses the essential current flow along the *z*-axis, which is illustrated by the red plane in b). The current density on the inner cavity walls is disturbed least by cutting the cavity along one of the green planes. In c) an actual cavity made out of aluminium is shown.

In [25] this problem is investigated to realise multilayer quantum circuits. They show that the cavity is allowed only to be cut such, that the induced currents in the cavity walls do not have to cross the seam. This is illustrated in figures 1.7 a) and b). A cut along any other direction leads to a dissipation of currents and thus to a decrease in the cavity's quality factor, especially when no indium is used to close the seam.

Quantisation

The oscillating currents in the cavity walls act the same as in an LC-circuit, where the charge difference *Q* in a capacitor interchanges with the magnetic flux Φ in an inductance. Therefore a single mode in the cavity is modelled by an LC-circuit with resonance frequency $\omega_c = \frac{1}{\sqrt{LC}}$. The total energy of an LC-circuit is given by the Hamiltonian

$$H_c = \frac{\Phi^2}{2L} + \frac{Q^2}{2C} \quad , \tag{1.21}$$



Figure 1.8: Circuit representation of a transmon in a) and a SQUID based transmon in b). The transmon is represented by a Josephson junction and a large shunt capacitance in parallel. Adding a second Jospehson junction in parallel to the first junction results in the SQUID based transmon. The SQUID based transmon is sensitive to an external magnetic flux Φ_{ext} through the enclosed area (green).

where the first term indicates the inductive and the second term the capacitive energy. One defines the canonical conjugated operators

$$\hat{\Phi} = i \sqrt{\frac{\hbar L \omega_c}{2}} (\hat{a} - \hat{a}^{\dagger}) \text{ and } \hat{Q} = \sqrt{\frac{\hbar C \omega_c}{2}} (\hat{a} + \hat{a}^{\dagger})$$
 (1.22)

which satisfy the quantisation relation $[\hat{Q}, \hat{\Phi}^{\dagger}] = i\hbar$ due to $[\hat{a}, \hat{a}^{\dagger}] = 1$. Inserting the operators $\hat{\Phi}$ and \hat{Q} into the Hamiltonian (1.21) gives

$$H_c = \hbar \omega_c \left(\hat{a}^{\dagger} \hat{a} + \frac{1}{2} \right) \tag{1.23}$$

the quantised Hamiltonian for an LC-circuit or microwave cavity.

1.4.2 The Transmon Qubit

The transmon consists out of two superconducting islands separated by a Josephson junction. The superconducting islands are designed such that the capacitive energy of the circuit is lowered. In a circuit representation, depicted in figure 1.8, a large shunt capacitance C_s is added next to the Josephson junction. As a result the total capacitance is given by $C_{\Sigma} = C_J + C_s$. The enhanced capacitance stabilises charge fluctuations and therefore increases the qubits coherence time.

Another version of the transmon has two Josephson junctions in parallel and forms a closed loop. A superconducting loop separated by two Josephson junctions is known as superconducting quantum interference device (SQUID). The SQUID based transmon design is sensitive to the flux that penetrates the loop. As a result the qubit's transition frequency becomes tunable by an external applied magnetic field.

Next the transmon is quantised. The results are then used to calculate the properties of a SQUID based transmon.

Quantisation

The transmon has a linear capacitive energy part and a non-linear inductive part. The sum of both gives the Hamiltonian

$$H_q = \frac{Q^2}{2C_{\Sigma}} - E_J \cos(\phi) \quad , \tag{1.24}$$

where E_i is the Josephson energy.

The transmon is an anharmonic oscillator and acts as a qubit when it is driven on its two lowest states only. As a consequence the cosine may be approximated for small phase differences. A Taylor series up to forth order yields

$$H_q \approx \frac{Q^2}{2C_{\Sigma}} - E_J + \frac{E_J}{2}\phi^2 - \frac{E_J}{24}\phi^4$$
(1.25)

such that the non-linear behaviour is still included in the last term. The first three terms approximate the cosine as a parabola, which describes a harmonic oscillator with a constant energy shift. The constant energy shift is neglected, because energy differences are measured.

Since the phase difference ϕ is connected to the flux variable Φ via $\phi = \frac{2\pi\Phi}{\Phi_0}$, the same operators \hat{Q} and $\hat{\Phi}$ from equation (1.22) are used to quantise the Hamiltonian. The only modification is that a different capacitance C_{Σ} and inductance $L_J = \frac{1}{E_J} \left(\frac{\Phi_0}{2\pi}\right)^2$ are introduced and the creation and annihilation operators are rewritten. Naturally this leads to a different resonance frequency $\omega_0 = \frac{1}{\sqrt{L_J C_{\Sigma}}}$ for the quadratic terms.

After inserting and rewriting the Hamiltonian (1.25) one obtains

$$\hat{H}_q \approx \hbar \omega_0 \hat{b}^{\dagger} \hat{b} - \frac{e^2}{24C_{\Sigma}} \left(\hat{b} - \hat{b}^{\dagger} \right)^4 \quad . \tag{1.26}$$

The last term is treated as capacitive energy that adds anharmonicity to the harmonic potential. One introduces the capacitive energy $E_C = \frac{e^2}{2C_{\Sigma}}$ and finds

$$(\hat{b}-\hat{b}^{\dagger})^4\approx 12\left(\frac{1}{2}(\hat{b}^{\dagger}\hat{b})^2+\hat{b}^{\dagger}\hat{b}\right)$$

by applying first order perturbation theory² and using the commutation relation $[\hat{b}, \hat{b}^{\dagger}] = 1$. The resulting Hamiltonian

$$\hat{H}_q \approx \hbar \omega_0 \hat{b}^{\dagger} \hat{b} - \frac{E_C}{2} \left(\hat{b}^{\dagger} \hat{b} \right)^2 - E_C \hat{b}^{\dagger} \hat{b}$$
(1.27)

introduces an energy shift $\hbar \omega_q = \hbar \omega_0 - E_C$.

²Expanding $(\hat{b} - \hat{b}^{\dagger})^4$ gives terms with a different number of \hat{b} and \hat{b}^{\dagger} that are multiplied with each other. Only terms with the same number of \hat{b} and \hat{b}^{\dagger} stay in first order perturbation theory due to energy conservation.

Next the number operator $\hat{b}^{\dagger}\hat{b}$ is replaced by the Pauli operator $\sigma_z = |e\rangle \langle e| - |g\rangle \langle g|$, because the transmon is driven only on its lowest two levels. Finally the Hamiltonian

$$\hat{H}_q = \frac{\hbar\omega_q}{2}\sigma_z \tag{1.28}$$

is found and describes the transmon as two level system.

The qubit's transition frequency is typically at some GHz. For the used quit in the final experiment $\omega_q = 3,931$ GHz is measured at the high frequency sweet spot. The anharmonicity is given by the capacitive energy and is typically few hundreds of MHz. The used qubit has an anharmonicity $E_C/\hbar \approx 300$ MHz.

SQUID Based Transmon

The SQUID based transmon is a superconducting loop with two Josephson junctions as depicted in figure 1.8 (b). Again it has a large shunt capacitance to avoid charge noise, which distinguishes it as transmon. The Hamiltonian is

$$H = \frac{Q^2}{2C_{\Sigma}} - E_{J1}\cos(\phi_1) - E_{J2}\cos(\phi_2)$$
(1.29)

Next one defines the phase difference $\phi = \phi_1 - \phi_2 = 2\pi n + 2\pi \Phi_{ext}/\Phi_0$, where *n* is an integer. The phase difference is related to an external applied flux Φ_{ext} through the loop. The effective phase difference $\varphi = (\phi_1 + \phi_2)/2$ and the total Josephson energy $E_{J\Sigma} = E_{J1} + E_{J2}$ are defined. A trigonometric transformation leads to

$$H = \frac{Q^2}{2C_{\Sigma}} - E_{J\Sigma} \left(\cos\left(\pi \frac{\Phi_{ext}}{\Phi_0}\right) \cos(\varphi) + d\sin\left(\pi \frac{\Phi_{ext}}{\Phi_0}\right) \sin(\varphi) \right) \quad , \qquad (1.30)$$

where $d = (E_{J2} - E_{J1})/(E_{J1} + E_{J2})$ is the SQUID's asymmetry parameter. One defines the constant phase shift $\tan(\phi_0) = d \tan(\pi \frac{\Phi_{ext}}{\Phi_0})$ and finds

$$H = \frac{Q^2}{2C_{\Sigma}} - E_{J\Sigma} \cos\left(\pi \frac{\Phi_{ext}}{\Phi_0}\right) \sqrt{1 + d^2 \tan\left(\pi \frac{\Phi_{ext}}{\Phi_0}\right)} \cos(\phi - \phi_0)$$
(1.31)

The form of this Hamiltonian is the same as in equation (1.24). The only difference is that the effective Josephson energy

$$E_J = E_{J\Sigma} \cos\left(\pi \frac{\Phi_{ext}}{\Phi_0}\right) \sqrt{1 + d^2 \tan\left(\pi \frac{\Phi_{ext}}{\Phi_0}\right)}$$
(1.32)

of the transmon is now tunable. As a result the transmon's transition frequency is tunable by an external magnetic field. The phase $\phi_0 \in [-\pi/2, \pi/2]$ only appears for an asymmetric junctions. It can be eliminated for a constant magnetic flux by a shift of variables. Quantisation then is the same as for a single junction transmon. In case of a time variable magnetic field the phase ϕ_0 offers an additional qubit control. The additional control leads to an additional qubit decay channel [22].



Figure 1.9: Circuit representation of the coupling between a transmon and a cavity. The cavity represented by an LC-circuit is capacitively coupled to the transmon.

1.4.3 Coupling A Transmon To A Cavity

To read-out the state of a transmon qubit one needs to couple it to a coherent microwave field in a cavity. As a consequence the system's Hamiltonian consists out of three parts like in the previously discussed cavity QED system. First, the microwave field is described as a single mode harmonic oscillator, see equation (1.23). Second, the transmon qubit acts the same as a spin-1/2 particle when it is driven on its lowest two levels, see equation (1.28). Finally, one has to consider the dipole interaction between the microwave field and the transmon. Adding all three parts together leads to the same mathematical description as in cavity QED. Therefore all Hamiltonians derived in section 1.3 are valid and QND measurements on the transmon are possible due to the dispersive Hamiltonian.

Figure 1.9 shows the coupling between a transmon and an LC-circuit in a circuit representation. The coupling constant

$$g = \frac{2\beta eV_0}{\hbar} \tag{1.33}$$

depends on the ratio $\beta = C_c/C_{\Sigma}$ between the coupling capacitance and the total capacitance $C_{\Sigma} = C_J + C_s + C_c$ [22]. Of course the gate voltage $V_0 = \sqrt{\frac{\hbar\omega_r}{2C_r}}$ is relevant for the coupling constant too.

Remarkably, all parameters in circuit QED can be designed more or less arbitrarily. Therefore high coupling strengths between cavity and qubit are possible, which is necessary for high contrast in the dispersive read-out.

1.5 Summary

The theoretical concepts of realising and controlling an artificial quantum bit have been shown. A Josephson junction, which is a tiny insulating gap between two superconducting islands, provides the required non-linearity to define a two-level system. Adding a large shunt capacitance in parallel to the Josephson junction enhances the stability of the two-level system. This configuration is then known as transmon. The transmon is expanded to the SQUID based transmon by adding a second Josephson junction in parallel. In this configuration the transition frequency is sensitive to the magnetic flux through the formed loop.

The transmon's state can be read-out by coupling it to a coherent microwave field. Therefore the transmon is enclosed into a microwave cavity, which is required because of two main reasons. On the one hand the cavity shields the transmon from noise and enhances the qubit's lifetime due to the Purcell effect. On the other hand the cavity makes dispersive read-out of the transmon's state possible.

The dispersive read-out is essential for a QND measurement. Originally the idea of a QND measurement comes from cavity QED experiments with Rydberg atoms. Due to the same physics the mathematics derived in section 1.3 is directly applied to circuit QED. All this effort of building and investigating circuit QED systems is motivated by performing quantum simulations in the next decade and realising a quantum computer in the future.

Experiments with superconducting qubits have been developed a lot since their start in the eighties [27]. The research on circuit QED systems has spread into many different directions, where a lot of problems have to be investigated and solved experimentally. One thing to solve is the fast flux control of SQUID based transmons in a three dimensional cavity architecture. There the main issue is to switch on a magnetic field next to a transmon, without decreasing its coherence time. The problem of getting a magnetic field inside the cavity is discussed in the following chapter.

Chapter 2

The Magnetic Field's Guide To The Cavity

2.1 Overview

This chapter shows how to guide a magnetic field into a conductive box. The magnetic field is used to control a SQUID based qubit, which is placed in the centre of the box. The box acts as a microwave resonator, which is referred to as cavity. The cavity has to be highly conductive to reach long qubit coherence times. Now the goal is to switch a constant magnetic field inside such a cavity on and off immediately to have fast control of the qubit frequency. This task, however, comes with some issues.

In the case of a standard design for a superconducting aluminium cavity, it is not possible to get any magnetic field from outside into the cavity due to the Meissner effect. Another option is to use a highly conductive but not superconducting material for building the cavity, like oxigen free highly conductive (OFHC) copper. Copper has a relative permeability close to one, which makes it practically not magnetisable. A magnetic field applied from outside can therefore penetrate the copper cavity completely without being deflected. But this is only valid for static or slowly changing magnetic fields below 1 kHz. In the case of fast changes above some kHz the field is attenuated strongly. The cause of the attenuation is the appearance of eddy currents trying to oppose the magnetic field change due to Faraday's law. The attenuation increases with higher frequencies dramatically. Above 100 kHz the magnetic field is attenuated by more than a factor of one thousand and it practically cannot penetrate the copper cavity any more.

There are two obvious options to circumvent these issues. First one could put a magnetic source inside the cavity close to the qubit. Such kind of solution is offered by flux bias lines [28]. Flux bias lines are a good solution for two-dimensional architectures and can be easily implemented as part of a coplanar resonator. In the case of a three-dimensional architecture it decreases the qubit lifetime, as it disturbs the cavity mode and acts as an additional port through which the cavity field can decay. Despite the capacitive coupling between the flux bias line and the qubit, the qubit decay can be prevented by filtering. However, since filtering is complex in this

case and never perfect, flux bias lines may not be an optimal solution. The second option is to put the magnetic source outside the copper cavity and try to compensate the effect of eddy currents. The compensation methods include cutting the cavity or applying very high currents. These solutions are not viable in this experiment.

Besides these two options, a third one is introduced in this chapter. A hose for magnetic fields is built to guide a magnetic field from the outside to the inside of a cavity. Based on the theory of transformation optics, a magnetic hose was first developed by [29] for static fields. A tiny topological change in the setup of the hose allows high frequency fields to pass through. As a result it can then be used to guide a high frequency magnetic field through a conductive wall or to send fast flux pulses from the outside of a cavity to a qubit inside.

Before introducing the magnetic hose, the chapter starts with detailing the generation of magnetic fields in general. Experimental aspects are taken into account step by step, leading to first limits in the realisation of fast flux pulses. The effect of eddy currents is investigated next. A theoretical model and measurements of the effect are discussed. The measurement results point out, that fast flux control of a qubit inside a cavity is not possible by simply putting a magnetic source outside the cavity. Finally the magnetic hose is introduced. The theory behind it is explained and first measurements are presented.



Figure 2.1: Conceptual flux bias line setup for fast flux control on a qubit. A SQUID based qubit is placed in the centre of a cavity. Next to the qubit a U-shaped wire is placed. This wire is a so called flux bias line. Switching on a current through the flux bias line generates a magnetic field. The field controls the magnetic flux through the qubit and thus the qubit's transition frequency.

2.2 Generating Magnetic Fields

The observation of a magnetic field is a pure relativistic effect. Any charged particle moving along a reference frame generates a magnetic field. In the final experiment described in this thesis a static magnetic field has to be switched on and off immediately. Therefore the generation of a static magnetic field is explained first, then the difficulties in switching it on and off immediately are discussed.

To produce the static magnetic field one needs a constant current of charged particles. The easiest way to realise this is to apply a constant voltage between two wire ends. As a result a constant current of electrons flows through the wire leading to a static magnetic field. Since the wire is in total neutral no electric fields appear. The wire is then put next to the qubit. Whenever a magnetic field is needed, the current through the wire is switched on. Figure 2.1 illustrates the idea in case of a flux bias line next to a SQUID based qubit enclosed by a cavity.

2.2.1 The Biot-Savart Law

The magnetostatic case is well described by Biot-Savart's law ([30] chapter 5.2). This law considers a given current density \vec{J} in a Volume V. The current density can have different values at different points \vec{x}' in this volume, but it has to be constant in time to be described by Biot-Savart's law. Any infinite small current density element acts as a magnetic field source. One has to notice, that the magnetic field at a point \vec{x} decreases with increasing distance $d = |\vec{x} - \vec{x}'|$ from the source. The decrease is proportional to $1/d^2$ in case of an infinitely small current density element. Integration along a given current density can then lead to specific solutions. Therefore the decrease depends strongly on the total current density's geometry. This is pointed out in figure 2.2. A magnetic field is a circular field perpendicular to the current flow. For this reason the cross product is part of Biot-Savart's law. Finally the



Figure 2.2: Comparison between different magnetic field source geometries. The compared geometries are sketched in figure a) to d). All geometries lie in the *xy*-plane and the field strength along the z-axis (red dot) is investigated. Figure (A) illustrates the magnetic field decrease along the distance *z* for a circular loop $B(z) = \frac{\mu_0 I}{2} \frac{r^2}{(r^2+z^2)^{3/2}}$ (blue), a square loop $B(z) = \frac{\mu_0 I}{\pi} \frac{4a^2}{(a^2+4z^2)\sqrt{2a^2+4z^2}}$ (green), a finite long rod $B(z) = \frac{\mu_0 I}{2\pi z} \frac{l}{\sqrt{l^2+4z^2}}$ (red) and an infinite long rod $B(z) = \frac{\mu_0 I}{2\pi z}$ (orange). The current through each geometry is set to $I = 2/\mu_0$ to be normalised with respect to the circular loop. Geometry values are set to r = 1, a = 2, l = 2 for fair comparison. In the limit of large distance the loop geometries behave like a dipole with the characteristic $1/z^3$ drop. Figure (B) illustrates the magnetic field decrease along *z* for circular loops of different radius. Again the current through each loop is set to $I = 2/\mu_0$ to be normalised with respect to the loop is set to $I = 2/\mu_0$ to be normalised normalised with respect to the current loops of different radius. Again the current through each loop is set to $I = 2/\mu_0$ to be normalised with respect to the loop with radius r = 1 (blue). Increasing the radius flattens out the magnetic field curve.

corresponding magnetic field in a specific point \vec{x} is given by

$$\vec{B}(\vec{x}) = \frac{\mu_0}{4\pi} \int_V \vec{J}(x') \times \frac{\vec{x} - \vec{x}'}{|\vec{x} - \vec{x}'|^3} d^3 x' \quad , \tag{2.1}$$

where the magnetic constant is defined as $\mu_0 = 4\pi \cdot 10^{-7}$.

In the experiments a wire of constant diameter is used to guide the current. Therefore two simplifications can be done. First the current density per wire length is assumed to be constant. In this case the current density is equal to a constant current I per wire length l and one can write J = I/l. Second the current is confined to the conductor and thus to a specific path in space. As a result the magnetic field at a specific point depends only on the amount of current through the wire and the wire geometry. Biot-Savart's law (2.1) is then simplified to

$$\vec{B}(\vec{x}) = \frac{I\mu_0}{4\pi} \int_l d\vec{l} \times \frac{\vec{x} - \vec{x}'}{|\vec{x} - \vec{x}'|^3} \quad .$$
(2.2)

The integration is along all infinitely small wire parts $d\vec{l}$, where each is placed at a point \vec{x}' in space. From this equation two important statements should be remembered:

- The magnetic field is direct proportional to the applied current through the wire. This makes magnetic fields easy to tune, in theory arbitrary values are possible.
- The magnetic field decreases with growing distance to the source. This decrease is in general given by the integral part of equation (2.2) and therefore depends on the path the wire takes in space, see figure 2.2.

2.2.2 General Fast Flux Limits

Biot-Savart's law is valid in the static case. It is used to calculate the magnetic field generated by a constant current through a given wire geometry. Here the interest is in switching a constant magnetic field on and off immediately to realise fast flux control on a qubit. Switching a magnetic field on and off means to switch a current through a given wire geometry on and off. To provide the required current, the wire is connected to a power supply and forms a closed circuit. The circuit is modelled by an RL-circuit, as depicted in figure 2.3. The resistance results from the finite conductivity of the wire and the inductance results form the wire's geometry. A capacitive part is neglected, as it plays only a minor role here. The switch-on process of an RL-circuit is investigated next.

When switching on a constant current through an inductance, the current does not jump instantly to its proposed value. This is due to Faraday's law. The switched on current generates a magnetic field and therefore a magnetic flux through the inductance. Any change of magnetic flux induces an electric field in the inductance. The induced field powers a current that tries to compensate the change in flux by building up a counteracting magnetic field. As a result the instantly switched on





Figure 2.3: Powered RL-circuit. The current *I* in the circuit depends on the voltage source and the resistance *R* only in the static case. The inductance *L* comes into play if V_0 changes in time, as in a switch-on process.

Figure 2.4: Faraday's law. A time variant magnetic field $\vec{B}(t)$ passes the area of a closed conducting loop and induces an electric field $\vec{E}(t)$ along the loop. The area is orientated normal to \vec{n} .

current will ramp up in a finite time. This ramp up is characterised by a time constant $\tau = L/R$. The time constant depends on the inductance's self-inductance *L* and the resistance *R* of the circuit. In the experiment it is required to have a low time constant to make fast switching possible.

The well known differential equation for the RL-circuit can be derived from Faraday's law¹ ([30] chapter 5.15):

$$\oint_C \vec{E}d\vec{l} = -\frac{d}{dt} \int_S \vec{B}\vec{n}da \qquad (2.3)$$

Faraday's law states, that an electric field \vec{E} along a closed circuit *C* is induced, whenever the magnetic flux through the surface *S* spanned by the closed circuit changes in time. The integral on the right hand side is defined as magnetic flux

$$\Phi = \int_{S} \vec{B}\vec{n}da \tag{2.4}$$

through *S*. Here \vec{n} is the normal vector perpendicular to *S*. The situation is shown in figure 2.4 for a single loop. Basically a change of flux is related to a change of magnetic field or a change of the surface in time. In general the surface *S* is equal to the surface spanned by the circuit *C*. Parts of the circuit are usually shielded or the flux through them is negligibly small. Therefore in most cases the only relevant surface in the circuit is the inductance's surface. This assumption is part of the following calculations.

¹Setting right hand side of equation (2.3) equal zero gives Ohm's law. Ohm's law is just a special case of Faraday's law.

If the area *A* of the inductance is invariant in time, the right hand side of equation (2.3) simplifies to

$$-\frac{d}{dt}\int_{S}\vec{B}\vec{n}da = -A\frac{dB_{\vec{n}}}{dt} \quad , \tag{2.5}$$

where $B_{\vec{n}}$ is the magnetic field component parallel to \vec{n} at the inductance's position. Now one can insert equation (2.2) for the magnetic field in equation (2.5). Since the path of the wire is fixed, only the current through the inductance depends on time. Therefore

$$-A\frac{dB_{\vec{n}}}{dt} = -A\frac{\mu_0}{4\pi}G_{\vec{n}}\frac{dI}{dt} \quad , (2.6)$$

where $G_{\vec{n}}$ is a geometric factor. This factor equals the integral in equation (2.2) scalar multiplied with \vec{n} and evaluated at the inductance's position. All factors in front of the time derivative are constant. Together they define the self-inductance² *L* of the inductance and one rewrites

$$-\frac{d}{dt}\int_{S}\vec{B}\vec{n}da = L\frac{dI}{dt} \quad . \tag{2.7}$$

The self-inductance always has a positive value. The minus sign vanishes, because \vec{n} points in a direction opposite to the magnetic field in case of self-induction³.

The left hand side describes the electric field along the circuit. Any voltage drop is added up, like in Ohm's law. The difference of the voltage source V_0 and the voltage along the wire's resistance V_R are the only relevant parts, such that

$$\oint_C \vec{E}d\vec{l} = V_0 - V_R \quad . \tag{2.8}$$

Putting left and right hand side together gives

$$V_0 - V_R = L \frac{dI}{dt} \iff V_0 = RI + L \frac{dI}{dt}$$
 (2.9)

To solve this differential equation an initial condition is needed. At t = 0 the voltage source is switched on to a specific value V_0 instantly. The instant voltage change leads to an instant current change in the circuit. The instant current change leads to an instant flux change through the inductance. Due to Faraday's law a counteracting voltage is induced. As a result there is no effective current in the circuit at t = 0. Therefore the initial condition is I(0) = 0 and

$$I(t) = \frac{V_0}{R} \left(1 - e^{-\frac{R}{L}t} \right) \Theta(t)$$
 (2.10)

²The self-inductance L is defined as

$$L = \frac{\mu_0}{4\pi I^2} \int d^3x \int d^3x' \frac{\vec{J}(\vec{x})\vec{J}(\vec{x'})}{|\vec{x} - \vec{x'}|}$$

following [30] chapter 5.17.

³The closed integral along the electric field and the closed integral along the current through the wire have opposite direction, meaning opposite signs. The normal vector \vec{n} is orientated corresponding the integration direction of the induced electric field. As a result it is anti-parallel to the self-induced magnetic field.



Figure 2.5: Process of switching on an RL-circuit. The current is normalised. The time is given in units of the time constant.

solves the differential equation. Here $\Theta(t)$ is the Heaviside function. It describes the process of switching on the constant voltage source. In case of a current source one writes $V_0/R = I_0$, where I_0 is the constant current output.

Equation (2.10) describes the behaviour of the current in the RL-circuit if a voltage source is switched on instantly. This behaviour is shown graphically in figure 2.5. The current cannot jump immediately to its proposed value. The time it takes to reach the final value is given in units of the time constant $\tau = L/R$. After 5 τ the current reaches more than 99% of its proposed value. Therefore the key for fast switching is a small time constant.

The time constant can be reduced in two ways. First one can add resistance to the circuit in series, second one can reduce the inductance. However, both options have disadvantages. In case of a finite voltage source additional resistance lowers the maximum current. Raising the resistance adds noise to the signal. Reducing the inductance is related to reducing the generated magnetic field. It is therefore essential to find a suitable balance between resistance and inductance.

2.2.3 Specific Fast Flux Requirements In The Experiment

In the experiment a few limits have to be taken into account. First the magnetic flux through the qubit has to reach at least one flux quantum Φ_0 for full tunability. Second there is a given distance between the magnetic field source and the qubit. This limit leads to a specific wire geometry for the magnetic source. Next the voltage source that powers the magnetic field source has a maximum output power. Finally the magnetic source's time constant has to be below 10 ns. Combining all of those requirements gives the relevant parameter range to design the final setup.


Figure 2.6: Schematic setup. A coil is placed outside a copper cavity. The generated magnetic field is used for qubit control.

Minimal Needed Magnetic Field

The field strength has to be chosen such, that a SQUID based qubit can be tuned over its full frequency range. Therefore the field has to be strong enough to thread a full flux quantum Φ_0 through the SQUID loop. A SQUID with an area *A* requires a maximum field $B = \Phi_0/A$. The larger the area the smaller is the maximum needed magnetic field. But larger SQUID based qubits suffer from flux noise [31]. Again the key is to find a suitable compromise. Typically areas are in the order of 0,01 to 1 mm^2 and fields between 2 and 200 nT are needed for full tunability. In the final experiment a SQUID with an area of $200 \times 200 \,\mu\text{m}^2$ is used. The required field is 52 nT for full tunability.

Optimal Magnetic Source Geometry

As shown in the previous sections the magnetic field decreases with increasing distance between source and probe. Thus it is very helpful to put the source as close to the probe as possible and it is beneficial to fit the wire geometry of the source to the probe dimensions. In the experiment a qubit is placed in the centre of a cavity. The source is expected to be outside the cavity, as depicted in figure 2.6. The setup implies a fixed distance between qubit and source in the range of some millimeters. The qubit is treated as a point like probe, because of its small size in comparison to the distance. In this case the optimal source geometry is a circular loop, which can be seen by the following argumentation.

To bring the source as close to the qubit as possible one is forced to put it on the outer cavity wall. The source is realised by a current through a wire. Therefore the wire has to form a two dimensional path along this wall. Since any electrical circuit is closed, the path of the wire has to be closed at some point and encloses an area. From equation (2.6) one can see, that the self-inductance is minimal for the smallest loop area and the shortest wire length. Both properties are fulfilled by a circular loop. For a given current a circular loop generates the highest magnetic field at the qubit's position with minimal self-inductance.

Minimising The Distance

The distance between magnetic source and qubit is given by the cavity parameters. Cavity walls are usually in the range of 5 to 12 mm thick. At the position of the source the wall thickness can be reduced to about 1 mm, if it is necessary. Below 1 mm the cavity walls get deformed or even brake during the manufacturing process. Additionally the cavity's inner volume is taken into account, where the qubit is placed in the centre.

The design of the cavity's inner volume is discussed in section 1.4.1. The fundamental mode's frequency has to be in the range of 9 GHz and higher modes should be far away from that frequency. Therefore in the experiment a size of $22 \times 22 \text{ mm}^2$ is used for the relevant inner cavity surface. So there are 11 mm left between the inner cavity wall and the qubit. All together the total distance between source and quibt can be minimised to 12 mm.

Voltage Source Limit

The used voltage source is an arbitrary wave generator (AWG) of model AOU-H3353 / H3354 from *Signadyne*. It has a maximal output voltage of $3 V_{pp}$ into 50Ω . Therefore it can source a current of 60 mA_{pp} maximally. The signal produced by the AWG is guided through a coaxial cable into the cryostat. On its way it has to pass a -20 dBm attenuator, which is necessary to get rid of thermal noise. The total connection has a resistance of approximately 70Ω for direct current. As a result the maximal current is reduced to about $4,3 \text{ mA}_{pp}$.

Adding additional resistance lowers the time constant. But it also lowers the maximal current. Since the current is already low, it is not preferable to add additional attenuation or resistance to the line.

Time Constant Limit

The aim is to switch on and off the current through the used magnetic source in a time below 100 ns. As shown before it takes about five time constants to reach nearly 100% of the desired current. The resistance of the circuit is given by the attenuator. From this value a limit for the maximal self-inductance can be calculated. Since $5\tau < 10^{-6}$ s fulfils present requirements the self-inductance *L* has to be smaller than $1,4\,\mu$ H.

Final Coil Dimensions

The used magnetic source should be strong enough to generate a field of 52 nT at a distance of about 12 mm. The distance includes the cavity's inner volume and the cavity wall thickness, which is assumed to be 1 mm for the used cavity. A maximal self-inductance of $1,4 \mu$ H is allowed. As argued before the best magnetic source geometry is a circular loop in this experiment. Now one needs to find a proper radius *r*.

The relevant mathematical equations are

$$B(z) = \frac{\mu_0 I}{2} \frac{r^2}{(r^2 + z^2)^{\frac{3}{2}}}$$
(2.11)

describing the loop's magnetic field along its symmetry axis and

$$L = \frac{\mu_0 r \pi}{2} \tag{2.12}$$

describing its self-inductance⁴. From this the minimal necessary radius can be calculated, which is about 7,5 mm assuming a current I of 4,3 mA and distance z of 12 mm. The related self-inductance is not larger than 15 nH. This value is allowed to be increased by nearly a factor of thousand. Therefore it is in principle possible to add additional loops and build a cylindrical coil. As a consequence less current is required for the same magnetic field strength.

The coil's magnetic field increases linearly with the number of turns if the distance between each loop is small compared to the distance between the coil and the probe, because each loop adds approximately a magnetic field given by equation (2.11) at the probe's position. The self inductance, however, considers the mutual inductance between each loop. Therefore the coil's self inductance increases quadratically with the number of turns.

2.2.4 So Far Conclusion

Until know it seems there are no problems to realise the experiment. Magnetic fields can be generated strong enough to control a qubit in the cavity from the outside. Even the time constant is small enough to reach nanosecond switching times. All in all the distance between source and qubit caused by the cavity is no challenge.

The big challenge in applying flux pulses comes with the cavity's conductivity. Using a superconductive cavity is no option, if one wants to get a magnetic field from the outside into the cavity. Due to the Meissner effect, no magnetic field can enter the superconductor. The solution is to use a non-superconducting but still highly conductive material for the cavity. The material of choice is copper, because of two main reasons. First, its conductivity is high enough such that the qubit coherence time is sufficiently long, about $100 \,\mu$ s. This time is necessary to successfully perform some operations in the order of 100 ns on the qubit. Second, copper has a relative permeability close to one and is therefore practically not magnetisable. Still the cavity is highly conductive and problems appear, when fast flux pulses try to penetrate the cavity wall.

⁴In this case it is not necessary to calculate the full magnetic field in every spatial direction. The qubit is very small compared to the coil and it is aligned with the loop's symmetry axis. Therefore calculating only the field along this symmetry axis is a sufficient and effective approximation. The self-inductance is calculated assuming the same field B(0) through all the loop's area.



Figure 2.7: Circuit model for eddy currents. A powered RL-circuit generates a time varying magnetic signal next to *n* passive RL-circuits. All passive RL-circuits are coupled among each other and to the powered RL-circuit via a mutual inductance M_{ij} .

2.3 The Eddy Current Problem

Whenever a magnetic flux change appears through a conductive surface, an electric field is induced in that surface. The induced electric field causes currents, so called eddy currents. Theses currents move in the conductive area such that, the external change in magnetic flux is compensated. Again this behaviour is explained by Faraday's law (2.3). The appearance of eddy currents grows proportional to the change of flux in time and thus leads to massive attenuation for high frequency magnetic fields. The flux caused by a magnetic square pulse changes instantly, so the eddy current effect will be maximal. As a result no fast flux pulses can be sent through the conductive cavity, without being distorted heavily.

2.3.1 A Theoretical Model For Eddy Currents

The problem of eddy currents is well known in NMR-systems. Reference [32] models eddy currents in a conducting surface by a series of RL-circuits. The resistance appears due to the surface's finite conductivity. The inductance comes from the effective path that the eddy currents take on the surface. For each conducting surface eddy currents are assumed to appear according to the surface geometry and orientation with respect to the applied magnetic field. To investigate such a system one writes down the differential equation for an RL-circuit for each eddy current that might appear in this configuration. The interaction among eddy currents is then investigated in the picture of RL-circuits. Figure 2.7 outlines the idea.

Each RL-circuit has a specific self-inductance L_i and resistance R_i and each RLcircuit is coupled inductively to all other RL-circuits via a mutual inductance M_{ij} , where $i \neq j$. One has to remember that the eddy currents appear only because a coil generates a magnetic pulse. As shown before a driven coil is modelled by an RL-circuit too. The coil has a self-inductance L_0 and is coupled to all eddy currents via a mutual inductance M_{0j} . These assumptions lead to a set of coupled differential equations

$$V_0 = R_0 I_0 + L_0 \frac{dI_i}{dt} + \sum_j^n M_{0j} \frac{dI_j}{dt}$$
(2.13)

$$0 = R_i I_i + L_i \frac{dI_i}{dt} + \sum_{j \neq i}^n M_{ij} \frac{dI_j}{dt}$$
(2.14)

describing the whole system⁵. The integer i ranges from 0 to n, meaning n eddy currents are part of the system.

It is not necessary to solve this set of differential equation for each current I_i exactly. The form of the first order differential equations gives some clue to the form of the solution. Reference [32] claims that

$$I_i(t) \propto -I_0 b_i e^{-t/\tau_i} \tag{2.15}$$

is valid for each appearing eddy current using signal processing theory for linear time invariant (LTI) systems. The factor b_i is dimensionless and has its origin in the interaction between all components. The time constant τ_i indicates the current's decay. All eddy currents contribute to an effective magnetic field of the form

$$B_e(t) \propto \sum_{i=0}^n -I_0 b_i e^{-t/\tau_i}$$
 (2.16)

This field counteracts the applied magnetic pulse. It appears almost instantly because the self-inductance of an eddy current is close to zero. Its decay is slow due to the high conductivity. From Faraday's law one can see the counteracting field B_e becomes even stronger for faster flux changes. The effect on an initial magnetic pulse might look like this:



If a magnetic square pulse is applied next to a conductive material, eddy currents in the conductive material attenuate the pulse. As a result the sharp edges are rounded.

2.3.2 Measuring The Eddy Current Effect

The strength of the eddy current effect can be easily measured. Figure 2.8 shows the setup. A signal generator is connected to a resistance and a coil in series. The coil is placed next to a disk made out of copper. On the other side of the disk a second coil is positioned, which is connected to a lock-in amplifier. The signal generator powers the first coil with a sinusoidal signal. Its frequency is varied from low to high in discrete steps. Due to the varying magnetic field generated by the first coil, a

⁵In [32] the applied magnetic pulses and conductive surfaces are orientated along specific directions in space. The ordering of eddy currents is then referred to the spatial orientations. As a result the set of differential equations has spatial symmetries that go along with their assumptions.



Figure 2.8: Setup to measure the effect of eddy currents on a time varying magnetic field. A signal generator powers coil 1. The coil generates a time varying magnetic field. This field is picked up by coil 2. An oscilloscope or lock-in amplifier measures the induced voltage in coil 2. In order to study the effects of eddy currents a conductive material like a copper plate is placed between coil 1 and 2.

voltage is induced in the second coil. The induced signal has the same frequency as the generated one. Therefore it can be detected by the lock-in amplifier, which gives amplitude and phase of the detected signal with respect to the signal sent by the first coil.

Figure 2.9 shows the measured eddy current effect on a sinusoidal magnetic field passing through a 1,5 mm thick copper plate of $45 \times 45 \text{ mm}^2$ in size. The illustrated frequency response shows three measurement series. The black series is measured without a copper disk between the two coils. This serves as reference for the other two series. The red series is taken at room temperature with a copper disk between the two coils. The green triangles show the same measurement series done in liquid nitrogen. One can clearly see the effect of eddy currents acting as a low pass filter of high order. The phase shift of more than 360 ° indicates a low pass filter of more than fourth order for the green series. At room temperature the cutoff frequency is close to 1 kHz. Lowering the temperature to 77 K increases the copper plate's conductivity. As a result the cutoff frequency is decreased and found close to 100 Hz. At 100 kHz the attenuation of the received magnetic field is -21,90(2) dB and -34,48(1) dB at 273 K and 77 K respectively⁶. Therefore the signal is attenuated more than a factor of one thousand at temperatures below 77 K.

At this point one faces two problems. First the fast flux pulse should be switched on and off within a few hundreds of nanoseconds. Looking at the measurement this seems not to be reachable. The magnetic field practically vanishes above 100 kHz which corresponds to a time scale of $10 \,\mu s$. Second the effect becomes even worse at lower temperatures. Experiments with superconducting qubits take place in a cryostat

⁶The decibel scale is a logarithmic scale to compare a power signal P(x) with a reference power P_0 . The power signal P(x) corresponds to a function $Q(x) = 10 \log_{10}(P/P_0)$ in decibel. When comparing a variable field strength F(x) with a reference field strength F_0 , the power ratio in decibel is given by $Q(x) = 10 \log_{10}(F^2(x)/F_0^2) = 20 \log_{10}(F(x)/F_0)$. Therefore the field strength in a frequency response plot corresponds to half of the drawn power value in decibel.



Figure 2.9: Frequency response of the transmitted magnetic field through a 1,5 mm thick copper plate. The black squares are a reference measurement without any copper next to the coils. The red dots and green circles are measurements with a copper plate between the coils at room temperature and in liquid nitrogen respectively. Horizontal lines are drawn at -3 dB and 45 ° to compare the data to a low pass filter. These lines define the filters cut off frequency. Measurements at frequencies below 5 Hz show a systematic error. Signals below about 60 dB are no more detectable.

with temperatures close to 10 mK. The low temperature raises the conductivity of copper and attenuation due to eddy currents become stronger. There are widely used solutions to circumvent problems with eddy currents, but they cannot always be applied to a given setup.

2.3.3 Solutions To Compensate Eddy Currents

Mainly there are two solutions to compensate eddy current effects. The first one tries to redirect the path where the eddy currents flow along. This is done by cutting the conducting surface apart. Eddy currents cannot flow across the introduced slits. Therefore they have to take another path, that is bounded by the slits. Figure 2.10 illustrates the idea. The second solution tries to counteract the magnetic field's attenuation by applying a corrected signal like this:





Figure 2.10: Two conducting blocks of the same size are shown in the sketch. The left block is untouched, the right one is cut into two pieces. A magnetic field is switched on. It penetrates both blocks in the *z*-direction (black circles with a dot). This causes the appearance of eddy currents (red dashed lines). Effectively all eddy currents form a current distribution (red solid lines) such that the sudden change of magnetic flux is compensated. For the cut block, eddy currents cancel each other close to to the slit.

As seen in figure 2.9 high frequency components are attenuated and phase shifted stronger. The corrected signal takes these changes into account. Thus high frequency components are amplified and phase shifted during generation. As a consequence the eddy currents modify the signal such that the desired one is applied.

Cutting The Conducting Surface

Slits in a conducting surface can reduce the eddy current effect. By increasing the number of slits the counteracting fields become smaller. To investigate this behaviour a simple setup is built. The transmission of a sinusoidal magnetic field through a copper plate like in figure 2.8 is measured. Now, the copper plate is cut into 1 mm thin slices at its centre. As a result slits are introduced in the plate, as depicted in figure 2.12. The number of slits is variable, since the single slices can be exchanged. Figure 2.12 shows the transmission through the copper plate for a different number of slits. Even a single slit enhances the detected flux by nearly a factor of ten at 1 MHz. The transmission is improved by additionally putting Teflon tape between the single parts of the copper plate. Teflon acts as an insulator and thus completely stops the flow of eddy currents between the touching parts. Furthermore the red lines are flatter than the the blue ones. At 20 MHz the transmission between the coils drops in general, as it can be seen from the reference measurement through air.

In the final experiment a magnetic pulse has to penetrate a conductive cavity. Eddy current effects can be reduced sufficiently by cutting the cavity apart many times. As shown in section 1.4.1, the cavity is allowed to be cut only along the direction of current distribution resulting from the field mode inside the cavity. However, as pointed out in [25], any introduced cut decreases the cavity's quality factor and should be avoided, despite the cut is along a valid path.



Figure 2.11: *SOLIDWORKS* sketch of a copper plate with a variable number of slices. The plate is cut into eight parts resulting in seven slits.



Figure 2.12: Measurement of the flux penetrating a copper plate with different number of slits. The black line is a reference measurement without any copper between or next to the coils. The blue lines show measurements for 1, 5 and 7 slits (from dark to light blue), the red lines show measurements for 1, 3 and 7 slits with isolating Teflon tape (from dark to light red).

Applying A Correction Signal

The second solution to compensate the effect of eddy currents is to generate a signal that takes the eddy current effects into account. Since eddy currents are modelled as RL-circuits, they behave like linear filters. One can in theory totally compensate this filtering by using signal processing theory in LTI systems [33], see appendix B. Reference [32] explains an algorithm to optimise the eddy current compensation based on signal processing theory in LTI systems.

As seen in figure 2.9 one needs about a factor of one thousand more current for high frequency components. In the final experiment this is not manageable. The used voltage source does not have the required power. One could think about amplifying the signal, but this introduces unwanted noise to the system and may heat the cryostat due to higher dissipation.

Despite there are two possible solutions to compensate the effect of eddy currents, both cannot be implemented in this experiment. Cutting the cavity leads to dissipation of the field inside the cavity and applying a corrected signal requires too high currents. The effect of eddy currents destroys the idea of controlling a qubit with fast magnetic pulses completely. Therefore a different way has to be found to solve this task. In the next section a new approach is investigated, that circumvents the problem with eddy currents.



Figure 2.13: Different magnetic hose designs. Figure a) shows the basic hose design for static fields. The grey core refers to a ferromagnetic material, the blue shell represents a superconductor. A magnetic field is guided along the *z*-axis through the hose. In b) the idea of improving the hose by raising the number of layers is visualised. Here there are four alternating layers. Figure c) illustrates a first design for a hose that guides high frequency magnetic fields along the *z*-direction. This is possible due to the applied slit. Therefore no counteracting eddy currents can flow in closed circles in the *x*-*y*-plane. In figure d) the slits are ordered in a different way to minimise magnetic field leakage. An insulating material (red) has to be put between each conductive layer in this configuration. Otherwise eddy currents are able to flow across the slits and form slightly deformed but closed circles in the *x*-*y*-plane.

2.4 The Magnetic Hose

A magnetic hose is a device that transfers magnetic fields through it. The working principle of such a hose is quite simple. Basically only two ingredients are needed, something that attracts magnetic field lines like a ferromagnetic material and something that expels magnetic field lines like a superconductor. The hose is realised by wrapping cylindrical layers of ferromagnetic material and superconductors alternating around each other. This is represented in figures 2.13a and 2.13b. A magnetic field applied at one end is then transferred to the other end.

The original design is obtained from transformation optics [34]. It is a technique to calculate magnetic permeability and electric permittivity that force electromagnetic fields along an arbitrary path in space. This theory got its main attention as solutions to cloaking and making things invisible were proposed [35]. Meanwhile experiments demonstrated first promising proposals and transformation optics is used in a wide range nowadays [36].

Here the technique is used to build a magnetic hose that guides a magnetic field into a superconducting box, the cavity. In this section the required theoretical tools are explained first. They are then applied to the idea of guiding a static magnetic field along a straight path. The resulting ideal magnetic permeability is approximately realised by a combination of materials. This metamaterial structure represents the magnetic hose for static fields. Finally a small modification from section 2.3.3 is applied to make the transfer of high frequency magnetic fields possible.

2.4.1 Transformation Optics

The idea of transformation optics is to guide electromagnetic fields along an arbitrary path in space. Electromagnetic fields change their propagation when they hit media



Figure 2.14: Sketch of the idea behind transformation optics. An electromagnetic wave (red) becomes deformed when entering a material with corresponding properties (blue).

with different magnetic permeability μ and electric permittivity ϵ . To guide them at will, material with very particular distributed μ and ϵ has to be placed in the space of interest.

The methode is to first calculate the required μ and ϵ distribution and then to find a material combination with the calculated properties. The procedure of calculation has a fixed algorithm and is described in the following.

- All starts in an empty Cartesian space. Empty means that μ(r
 ⁱ) = μ₀1 and ε(r
 ⁱ) = ε₀1 in any spatial directions r
 ⁱ. Here 1 is the three dimensional unitary matrix, μ₀ and ε₀ are the magnetic and electric constant respectively. In this empty space electromagnetic waves travel along straight lines as depicted in figure 2.14a.
- Now one deforms the space of interest in such a way, that in the deformed space electromagnetic fields behave as desired. This is done by a spatial transformation

$$\vec{r}' = \vec{r}'(x, y, z) \iff \begin{cases} x' = x'(x, y, z) \\ y' = y'(x, y, z) \\ z' = z'(x, y, z) \end{cases}$$
 (2.17)

on purpose. With respect to this new coordinate system straight rays in the Cartesian space are described as curved rays. On the other hand rays that move straight in the new coordinate system are curved rays in the Cartesian space, see figure 2.14b.

 The last step is to calculate the relevant μ'(r
') and ε'(r
') tensors for bending the fields in the space of interest. The tensors are given by

$$\mu' = \frac{\Lambda \mu_0 \Lambda^{\mathrm{T}}}{\det \Lambda} \quad \text{and} \quad \epsilon' = \frac{\Lambda \epsilon_0 \Lambda^{\mathrm{T}}}{\det \Lambda} \quad ,$$
 (2.18)

where

$$\Lambda_{ij} = \frac{\partial \vec{r}'_i}{\partial \vec{r}_j} \tag{2.19}$$



Figure 2.15: Representation of the spatial transformation for the magnetic hose calculations. In a) two points, z_1 and z_2 , along the *z*-axis are selected. Between these two points the whole space, except for an infinitely small part ξ , is cut away in b). The missing space is replaced in c) by shifting $z_1 + \xi$ to z_2 and stretching the space between z_1 and ξ apart.

is the Jacobian transformation matrix. These transformations are valid, because Maxwell equations are invariant under spatial transformation. The result is illustrated in figure 2.14c.

After these calculations another task comes up. One has to find a way to realise the materials with relevant permeability and permittivity calculated in 2.18. Typically μ' and ϵ' are distributed anisotropic and highly inhomogeneous. A realisation of such a material is not always feasible or only approachable for special cases.

Calculating The Magnetic Hose

First⁷ one considers an empty Cartesian space. Then a transformation has to be done that transfers magnetic fields. The idea is to cut out the space between two points z_1 and z_2 except for an infinitely small part of thickness ξ . The boundary $z_1 + \xi$ is shifted to the position of z_2 . Simultaneously the part of thickness ξ is stretched to connect the space between z_1 and z_2 again. The transformation

$$z' = z,$$
 $z \in (-\infty, z_1) \Leftrightarrow z' \in (-\infty, z_1);$ (2.20)

$$z' = \frac{z_2 - z_1}{\xi} (z - z_1) + z_1, \qquad z \in [z_1, z_1 + \xi) \iff z' \in [z_1, z_2); \tag{2.21}$$

$$z' = z + (z_2 - z_1 - \xi), \qquad z \in [z_1 + \xi, \infty) \iff z' \in [z_2, \infty)$$
(2.22)

describes this process. It is represented in figure 2.15. The orthogonal coordinates x' and y' are unaffected.

Next the Jacobian matrix and the resulting μ' is calculated. The result is given by

$$\mu' = \mu_0,$$
 $z' \in (-\infty, z_1);$ (2.23)

$$\mu' = \mu_0 \cdot \operatorname{diag}\left(\frac{\xi}{z_2 - z_1}, \frac{\xi}{z_2 - z_1}, \frac{z_2 - z_1}{\xi}\right), \qquad z' \in [z_1, z_2); \quad (2.24)$$

$$\mu' = \mu_0, \qquad z' \in [z_2, \infty)$$
 (2.25)

⁷The calculations and results are taken from the supplementary of [29].

in the whole space. To shift magnetic field form z_1 to z_2 one has to put a material with the calculated properties between the two points. In the limit $\xi \to 0$ a material with $\mu_z = \mu'_{z'z'} \to \infty$ along the *z*-axis and $\mu_\rho = \mu'_{x'x'} = \mu'_{y'y'} \to 0$ along the radial direction is needed.

The required material properties are known. But no material with such properties is known. Therefore an approximation is done by combining two known materials. Ferromagnetic materials have a large permeability and attract magnetic fields. Superconductors completely repel magnetic fields. The solution is to alternately wrap layers of ferromagnetic and superconducting material in a cylindrical form, like in figure 2.13b. In this combination the ferromagnetic material provides a large permeability in *z*-direction and the superconductor sets μ_{ρ} effectively to zero. The higher the density of alternating layers the better is the approximation to the ideal material. Reference [29] predicts that even a few layers achieve a significant transport of magnetic field. They could also show the working principle of such a hose for static fields. In the case of high frequency fields a tiny modification has to be done.

2.4.2 A Magnetic Hose For High Frequency Fields

Sending a high frequency field through a magnetic hose described in figure 2.13b does not work. Eddy currents will flow in closed circles in the xy-plane and counteract the magnetic flux change through the hose. But, there is an easy way to prevent eddy currents in the xy-plane as shown in section 2.3.3. Cutting each layer along the z-axis like in figure 2.13c prevents the eddy currents from flowing in a closed circle in the xy-plane.

Even though the introduced cut prevents the counteracting eddy currents, magnetic field might leak through the resulting slit. Therefore, it is necessary to alternate the position of a slit with respect to each layer. As a consequence the leakage is minimised. However, alternating the slit position makes conductivity between layers possible. As a result eddy currents appear again. Thus the final solution is to put an insulating layer between the conducting layers. Figure 2.13d illustrates the design for a high frequency magnetic hose.

The magnetic hose is tested first in a slightly different way. Instead of superconducting layers, layers of copper are used. Copper acts as a diamagnet⁸. It does not perform as perfect as a superconductor, but the property enhances for high frequency fields. This facilitates tests at room temperature. Thus measurements can be done in a quite simple setup without the need for a cryostat.

In the final experiment such a hose should guide a magnetic field from the outside to the inside of a conducting cavity. Therefore the first measurements test if the hose is working in general and if a magnetic field can be transmitted through a copper disk. Finally it is tested weather a magnetic field can be sent into a copper cavity with a hose made out of copper at room temperature.

⁸Ironically the diamagnetic behaviour is based on eddy currents.



Figure 2.16: Different measurement setup with the magnetic hose and three different copper disks. Setup a) is used as reference measurement. No components are placed between coil 1 and coil 2. The distance between the two coils is the same in all measurement setups. This is necessary for comparison. In a)-d) three different copper disks are put between the coils: an untreated one in b), one with a hole in the center in c) and one with a hole and a slit from the centre to the edge in d). Measurements f) to h) show the configurations with hose and disk. The distance scale in g) and h) relates to the part of the hose that is pushed through the hole.



Figure 2.17: Frequency response of different setups with the self-made magnetic hose and three different copper disks. The results are related to the measurement setups in 2.16 in the following. Black squares represent the reference measurement between the two coils only (2.16a). Red circles refer to 2.16e, purple upside triangles to 2.16b, magenta downside triangles to 2.16f, dark blue diamonds to 2.16c, light blue left side triangles to 2.16h, dark green right side triangles to 2.16d and light green hexagons to 2.16h. The measurements in air and through the slit copper disk coincide. Adding the hose amplifies the signal in both setups but does not change the phase. The solid disk and the copper disk with a hole show the same behaviour in amplitude and phase.

Measurements With A Self-Made Magnetic Hose

First measurements are taken with a self-made magnetic hose. A steel drill-bit with a diameter of 2,5 mm acts as cylindric core. Around that, copper tape and steel tape are wrapped. Both layers are connected with adhesive tape, which acts as an insulator. In total, four alternating layers make the hose 7,5 mm thick in diameter. The length is 26,3 mm. With this hose the magnetic field transfer between two coils is measured. All different measurement setups for the first series are illustrated in figure 2.16 and the corresponding results are presented in figure 2.17.

One can see a clear improvement if the hose is between the two coils. The signal is amplified and the phase stays nearly constant. For frequencies higher than 10 kHz the signal strength becomes even stronger. This is related to the increasing diamagnetic property of copper with increasing frequency.

Next a 2 mm thick copper disk with 12 cm diameter is put between the two coils. As expected one observes an attenuation and a phase shift for high frequencies. Drilling a hole with 7,6 mm diameter into the disk makes no difference. The result is the same as if there was no hole. Cutting a 0,3 mm thin slit from the edge to the central hole of the disk changes the behaviour totally. Eddy currents are prevented



Figure 2.18: Frequency response of different setups with the self-made magnetic hose and the copper disk with a hole. Black squares show the reference measurement in air between the two coils and red circles represent the measurement with the hose between them. The other measurement series are measured with the setup shown in 2.16g, where the hose is pushed through the disk's hole. The fraction is related to the part of the the hose that is pushed through.

and the related attenuation vanishes.

The three setups with copper disks are connected with the hose. If the hose is placed close to the untreated copper disk, the signal is amplified but shows the same attenuation and phase shift. In the case of the disk with a slit, again amplification is observed and no phase shift appears. Finally, the maybe most relevant measurement is, when the hose is put through the hole in the copper disk. This combination comes close to the main idea of guiding a magnetic field into a cavity. In this configuration a resonance appears. It is investigated in a further measurement series in the following.

The hose is put through the holed Cu-disk for different lengths, shown in figure 2.16g. If one hose end is put asymmetric into the copper disk, the resonance is shifted to higher frequencies and the attenuation is stronger, see figure 2.18. The resonance for the symmetric setup is measured more precisely, as it can be seen in figure 2.19. A phase shift of $180,4(2,4)^{\circ}$ is measured at 2,153(5) kHz, where the received signal power drops to a minimum. This indicates a resonance effect.

The cause for this behaviour is a combination of three effects. First, there are eddy currents in the copper disk attenuating the signal. These eddy currents mainly result from magnetic field lines that leak through the copper shells of the hose. Second, at higher frequencies the shielding increases and less magnetic field lines leak through the hose layers. Third, some magnetic field lines that pass the hose cannot close their way back through the hose. They have to pass the copper disk surrounding the hose. If one hose end is close to the copper disk, the field line density through the disk



Figure 2.19: Accurate frequency response of transmission through the self-made hose symmetrically placed in the copper disk with hole. The reference measurement for the magnitude is the measurement series through air in figure 2.18. The resonance frequency is found at 2,153(5) kHz by fitting the tip of the tilted peak with a Lorentzian curve. The phase signal is fitted with two straight lines through points next to its inflection points. The distance between the points, where the fitted lines cross the resonance frequency, results in a 180,4(2,4) ° phase shift.

next to the hose is higher. Therefore the attenuation due to eddy currents is stronger. This results in a lower maximal signal strength for frequencies above the resonance.

Measurements With A Fabricated Hose

Since the first measurements with the self-made hose are promising to transport a magnetic field into a cavity, the subsequent step is to improve the magnetic hose. The next hoses are fabricated by the local workshop in a well controlled way. The manufactured layers are about $200 \,\mu$ m thin. This is the minimal thickness that is possible to manufacture in the workshop. Thinner layers break during the wire erosion process. Figure 2.20 shows the CAD-drawing of the fabricated hose. A steel core with 1 mm diameter is used as core. Then alternating copper and steel layers are wrapped around. In between every second layer is Teflon tape for isolation. The outermost shell and the steel core can be removed and varied between three different lengths. The three available shells and cores are 20 mm, 25 mm and 30 mm long. The inner parts are fabricated. They have different diamagnetic material and number of layers. Two use copper as diamagnet, and the other two are made out of aluminium. The copper ones are intended for measurements at room temperature, the aluminium ones are for measurements in the cryostat.



Figure 2.20: *SOLIDWORKS* sketch of a magnetic hose fabricated by the workshop. The left picture shows the frontside. Four copper layers can be seen, represented in brown colour. In between two copper layers is a steel layer (light grey) and a Teflon layer (dark grey). In the right picture the larger outermost shell can be seen. Inside one overlapping end a coil is placed to generate a magnetic field.

The following measurements are taken at room temperature. Therefore the two copper hoses are used. One hose has three alternating layers with a diameter of 3,6 mm and the other one has four layers with a diameter of 4,4 mm. The outermost shell of 30 mm is chosen for both hoses⁹. It is 10 mm longer than the inner part and therefore it sticks 5 mm out at each side, as illustrated in figure 2.20. Again two coils are used to measure the frequency response like in figure 2.17a. Coil 1 generates the signal and coil 2 receives it. Both coils have ten turns, a diameter of 3 mm and a length of about 7 mm. To put coil 1 as close to the inner part as possible it is placed into the outermost shell, like it is sketched in figure 2.24. Coil 2 is put at the end of the complete hose as usual.

First measurements investigate the difference in layer numbers and length of the steel core. Figure 2.21 shows a measurement from 100 Hz to 100 kHz. It is recorded with a lock-in amplifier. A network analyser is used for the measurements in figure 2.22. There the signal spans from 100 kHz to 300 MHz. One can clearly see, that the transmitted magnetic field increases with the number of layers. But a longer iron core enhances this effect much more. The phase varies in the range of $\pm 10^{\circ}$ with respect to the mean of a single measurement series. Again the hose increases its performance at higher frequencies up to about 100 kHz. At higher frequencies the transmitted signal decreases. In the range of 30 to 40 MHz there is a drop in transmission. This drop is related to the used coils, because it is also visible in the reference measurement without hose. Finally above 60 to 70 MHz strange things happen. The signal is totally messed up. A possible explanation might be, that in

⁹The hose is structured like a coaxial cable. Therefore electromagnetic waves are able to transfer through the hose too. Connected to the cavity, the hose offers an additional channel for microwaves to get from the inside to the outside of the cavity or the other way round. As a result the internal quality factor of the cavity decreases. To prevent microwaves from passing the hose, the outermost shell is made longer than the inner part. This acts as a circular waveguide and attenuates microwaves below its cutoff frequency $f_c = \frac{1.8412c}{2\pi r} \approx 50$ GHz considering an inner radius *r* of 1,75 mm at the speed of light *c* in vacuum. Below the cutoff the attenuation in Np/m is equal to the complex propagation constant $i\beta = \sqrt{(\frac{2\pi f}{c})^2 - (\frac{1.8412}{r})^2}$, which is equivalent to approximately 9 dB/mm for f = 9 GHz [23].



Figure 2.21: Frequency response of the fabricated hose in the range of 100 Hz to 100 kHz. Black squares represent a reference measurement through air. Measurements with hoses made out of three copper layers with short (red dots) and long iron core (green upside triangles) as well as measurements with hoses made out of four copper layers with short (dark blue downside triangles) and long iron cores (light blue diamonds) are done.

this regime resonances between the two coils and within the hose appear.

The longer iron core of 30 mm length does improve the signal up to 10 MHz. From then on it matches the signal of the shorter iron core of 20 mm length. Since a flat frequency response is preferable, the short iron core is taken for further investigations. The larger hose with four alternating layers has better transmission than the hose with three layers. The problem is that it needs more space resulting in a stronger modification of the cavity. A larger hose provides a better connection for microwaves to the outside of the cavity, which is unwanted in the final experiment. Therefore the smaller hose with short iron core is chosen for further investigations.

In the next measurement series a copper disk with hole is added. The measurements are represented in figure 2.23. First a reference measurement is recorded. Then the thinner copper hose with three layers and short iron core is measured again. This hose is then pushed through the copper disk. Once it is put half through and once it is flush with the copper disk. At the end of this measurement series again a reference measurement is done, because the position between the coils moved a bit when putting the copper disk into the setup. The reference value is given by the mean of both measurements through air in the range of 1 to 10 MHz. If the hose is pushed half through the disk, the transmission is still better than in air. This gives hope that guiding a magnetic field through a hose into a cavity works, which is treated next.



Figure 2.22: Frequency response of fabricated the hose in the frequency range of 100 kHz to 300 MHz. The black line in the range of 1 to 10 MHz is used as reference measurement through air. The colour code for the other measurements with different hoses is the same as in 2.21.



Figure 2.23: Frequency response of the fabricated hose with the copper disk in the range of 100 kHz to 300 MHz. The mean value of the black and grey line in the range of 1 to 10 MHz is used as reference measurement through air. The red line shows a measurement with the hose only. Green and orange lines are measurements of the hose in a copper disk pushed half through or flush respectively.



Figure 2.24: Setup for measurements with a magnetic hose and a copper cavity. Coil 1 is placed inside the outermost shell of the hose. The other end is flush with the inner cavity wall. Coil 2 is placed in the cavity centre.

Meausrements With A Magnetic Hose And A Copper Cavity

Two things have to be investigated now. First if it is possible to get any magnetic field inside the cavity and second if the hose has some unwanted effects on the cavity mode.

To measure magnetic field transmission again two coils are used. Coil 1 generates the field. It is placed inside the hose's outermost shell as close to the inner part as possible. Coil 2 is fixed in the cavity centre, where usually the qubit is placed. Both coils have ten turns and the diameter is about 3 mm. The hose is attached to the cavity, as depicted in figure 2.24. The part of the hose entering the cavity is flush with the inner cavity wall. In figure 2.25 and 2.26 the corresponding frequency response is illustrated.

The detected signal in this measurements is very low, as there is additional space between the hose and the receiving coil of about 9 mm. It was hard to even detect the reference signal through air. Additional amplification and averaging is necessary to detect the small signals. Since the reference signal is not that flat, it is not used for calculating a relative dB-scale. Therefore the absolute measured magnetic flux is shown.

In the frequency response form 100 Hz to 100 kHz (fig. 2.25) the signal is attenuated strongly for higher frequencies. The behaviour of the phase looks quite strange. As it is expected to be flat as the green triangles for all measurements. One explanation for this behavior is that the phase in the other measurements was locked close to a 180° phase shift. Averaging over a lot of measurements leads to the large jumping for small deviations.

The frequency response from 100 kHz to 10 GHz (fig. 2.26) is quite different from the former one. All three signals start at the same level. Then there is a linear reduction in signal until 4 MHz. From then on the transmission through air stays constant, but the transmission to the cavity increases. A reference slope is drawn in light blue. This rise may result form the diamagnetic behaviour of the copper. For increasing frequency the shielding effect of copper is enhanced. The magnetic field that enters the cavity through the hose cannot leave the cavity through the inner cavity walls.



Figure 2.25: Frequency response of the magnetic hose flush in the copper cavity from 100 Hz to 100 kHz. The black squares show a measurement with the hose only. Both coils and the hose are kept at the same distance as in the measurements with the cavity. Two measurements with cavity are done. One at room temperature (red circles) and another one in liquid nitrogen (green triangles).



Figure 2.26: Frequency response of the magnetic hose flush in the copper cavity from 100 kHz to 10 GHz. The colour code for the represented results is the same as in figure 2.25. A light blue reference line is drawn to compare the rise of the transmitted magnetic field. Its slope has 100 Vs per decade.



Figure 2.27: Quality factors of an undercoupled and overcoupled copper cavity with hose. The measurements are taken at room temperature. Blue lines show the results from the overcoupled and red lines from the undercoupled setup. The quality factors Q_c and Q_i are presented by squares and triangles, respectively.

The measurement clearly shows, that high frequency magnetic fields can be sent into a cavity. There is some attenuation for low frequencies, but it is compatible. For better transmission the hose can be pushed more inside the cavity. This will shift the cavity's resonance frequency to a higher frequency, as the inner cavity volume is reduced. Furthermore the hose might influence the quality factor, which is investigated next.

The cavity has two ports. Each port can be used to send microwaves into the cavity or receive microwaves from the cavity. In this configuration the scattering parameters can be determined. For the following measurements only one port is used to measure reflection, which is the S_{11} parameter. Thus a microwave signal is sent to one port of the cavity. All reflected signals that cannot enter the cavity are detected. The second port is closed. From performing a reflection measurement the cavity's resonance frequency and quality factors can be determined.

Reflection measurements are done for two different pinlengths of the microwave port. The pinlength of the port specifies the coupling between the port and the cavity. If the pin enters the cavity, it is overcoupled. A longer pin means a stronger coupling and the microwave field enters the cavity more easily. If the pin does not enter the cavity, the cavity is in the undercoupled regime. The pinlength determines the coupling quality factor Q_c of the setup.

Figure 2.27 shows how the cavity quality factors change by pushing the hose more inside the cavity. The coupling quality factor Q_c does not decrease in both cases, the undercoupled and the overcoupled one. The internal quality factor Q_i decrease by pushing the hose more inside the cavity. Since $1/Q_l = 1/Q_c + 1/Q_i$, the

load quality factor Q_l follows the internal one. This indicates internal loss due to the hose. Additionally the cavity frequency is shifted maximally by about 30 MHz in both cases.

The hose has an influence on the cavity's internal quality factor and its resonance frequency. By pushing the hose deeper inside the cavity, internal losses are enhanced and the cavity's resonance frequency is shifted to higher frequencies. However, there is no influence on the cavity, if the hose is flush with the inner cavity wall. Guiding a fast oscillating magnetic field inside a cavity has been achieved and it is feasible without additional influence on the cavity.

2.5 Summary

It has been shown experimentally that it is possible to guide a high frequency magnetic field into a conductive box. Magnetic field transfer to the cavity is achieved by using a magnetic hose. In section 2.2.3 experimental limits are discussed without considering the hose. The discussed limits stay the same, but the experimental setup is changed. A magnetic hose is used to transfer the magnetic field from the outside to the inside of the cavity, such that a SQUID based qubit inside the cavity can be controlled. Still a field of 52 nT is needed for full flux control on the qubit and the coil's self-inductance has to be below $1,4 \,\mu$ H.

The actual hose design in figure 2.20 suggests to put a coil inside one of its ends. Since the outermost shell has an inner diameter of about 3,3 mm, the coil has to be a bit smaller. To enhance the generated magnetic field the final coil has ten turns. The resulting self-inductance is about 0,3 μ H. The resulting field inside the cavity can be estimated by the measurement depicted in figure 2.25. There the magnetic flux through a coil with ten turns and a diameter of 1,5 mm is shown. A minimal flux of 10^{-12} Vs is detected at room temperature and corresponds to a magnetic field of 14 nT. This is a too small field. The value becomes even smaller when the current of 333 mA through the coil is compared to the maximal current of 43 mA_{pp} that can be supplied by the AWG. As this is the worst case scenario, there are some ways to improve the field strength.

First of all, the performance of the hose improves at lower temperatures. This is shown in figure 2.25 for a hose that uses copper as diamagnetic material. In case of a superconducting material the field transfer is expected to improve even more.

A main improvement is achieved by putting the hose as close to the probe as possible. In the previous measurements the hose is put into the cavity, flush with the inner cavity wall. So there is a lot of unnecessary space between probe and hose. Using a hose with a superconducting outermost layer should not have a noticeable effect on the cavity's internal quality factor and thus the hose can be put close to the qubit.

As shown in figure 2.17 a single slit into a copper disk prevents the formation of eddy current. It is possible to cut the cavity along specific lines without losing much of the cavity's internal quality factor, see figure 1.7. Combining such an additional cut with the hose might enhance the magnetic field transport a lot. This is an easy option, but the benefits have not been investigated so far.

The performance of the hose increases when a material with high permeability is used. Here steel is used as ferromagnetic material inside the hose. The relative magnetic permeability of the steel used in the experiment is unknown. One knows for sure that it has to be much lower than the permeability of pure iron. Still there are materials with much higher permeability than pure iron available, like mu-metal. This could be used instead of steel to improve the magnetic field transport, because the magnetisation is much better.

Magnetic field lines have to be always closed. The hose is not closed. Thus some field lines may form closed loops all inside the hose. All others have to close by going around the hose. These field lines are connected to eddy current effects, if conducting material surrounds the hose, like a cavity. In case of a cavity it may be the best to built a hose that enters the cavity at two opposite sides and connects them. In this case the coil has to be embedded in the hose. Otherwise the hose can be cut into two parts that connect to the coil.

From these measurements a realisation of fast flux control of a SQUID based qubit in a 3D cavity architecture seams feasible. The results are promising and there are a lot of possibilities to improve the setup. The next step is to test the setup in the cryostat on a qubit.

Chapter 3

Fast Flux Pulses

3.1 Overview

In this chapter the transition frequency of a SQUID based transmon qubit is switched by a magnetic pulse. The qubit is placed in the centre of a microwave cavity and the magnetic pulse is generated by a coil outside the cavity. A magnetic hose is put into the cavity. The hose provides a connection for magnetic fields between the outside and the inside of the cavity. Thus a fast flux pulse applied form the outside is able to change the transition frequency of a qubit inside the cavity.

In the beginning of this chapter the experimental setup is presented. Next, first measurements on the qubit and cavity are done to characterise the system. Before performing experiments with fast flux pulses, the generation of current pulses is discussed theoretically. The current pulses are provided by an AWG with a sample rate of 200 MS/s. The bare output of the AWG pulses are investigated and adjusted.

Finally the fast flux pulses are applied to the qubit. The experimental results show that a flux pulse is able to tune the qubit from one to another transition frequency within 200 ns.



Figure 3.1: Cavity setup with hose and two qubits. The hose is about 2 mm away from the central qubit when the cavity is closed.

3.2 Characterising The Qubit

The following two subsections present the measurement setup and how a single qubit is characterised. The heart of the setup is the cavity with qubit and hose, which is all placed in a cryostat. Next the connection from the outside of the cryostat to the cavity is explained. The setup for exciting and detecting the qubit is presented. In the following first measurements are done on the cavity and qubit to characterise the system. All essential parameters result from these measurements.

3.2.1 Measurement Setup

Heart Of Setup

A SQUID based transmon is placed in the centre of a cavity. The cavity has an inner volume of $22 \times 22 \times 10 \text{ mm}^3$, resulting in a resonance frequency of 9,6 GHz approximately. The transmon is designed to have a tunability of 1 GHz around its transition frequency. The transition frequency should be close to 6,9 GHz. The cavity frequency, the qubit transition frequency and their coupling are chosen such that the dispersive limit is reached.

The transmon's SQUID loop has an area of $200 \times 200 \,\mu\text{m}^2$ for sufficient magnetic flux control. Besides that, the transmons anharmonicity should be in the range of 100 to 200 MHz and the dispersive shift should be around 5 MHz for sufficiently good read-out. The ratio E_J/E_C is designed to a value of 70 to make sure the qubit operates in the transmon regime and the anharmonicity is in the proposed range. The values are measured to verify if the qubit is built as intended.

The hose is made out of aluminium, steel and Teflon with three alternating layers, described in section 2.4.2. Inside the hose there is a coil made out of a 279 μ m thin superconducting wire. The coil has a mean diameter of about 2,5 mm and a length of

3,7 mm approximately. The coil is soldered to an SMA connector, which is attached to the cavity. Outside the cavity walls there are two indentations for two larger coils. The larger coils are made out of a $152 \,\mu$ m thick superconducting wire. The wire is wound forty times around a cylindric copper body, that is 2 mm in diameter. The cavity is shown in figure 3.1. It is mounted below the 20 mK plate in the cryostat and surrounded by an Al-shield to protect the system from external magnetic fields.

Connection To The Cryostat

The cavity has two SMA ports to measure the transmission. The input port is undercoupled to get microwave signals into the cavity and the output port is overcoupled to read-out the transmission. Before entering the cryostat, the input signal has to pass a DC-block on top of the cryostat, to get rid of any DC-offset. The DC-block is connected to a 20 dB attenuator between the 70 K and 4 K-plate. Next the signal is guided into a 30 dB attenuator between the 100 mK and the 20 mK-plate. All this attenuation is necessary to reduce thermal noise. Still between the two lowest plates, an Eccosorb filter absorbs infrared radiation that appears in the signal path. Finally, the signal enters the cavity.

The cavity's output is connected to a DC-12 GHz bandpass filter followed by two isolators between the 100 mK and the 20 mK-plate. An isolator allows a signal to pass through in one direction only. Therefore practically no signal enters the cavity through the output port. The output signal is guided into a high-electron-mobility transistor (HEMT) for first amplification between the 70 K and 4 K-plate. Then the signal leaves the cryostat and is amplified by 40 dB classically at room temperature before it is recorded.

The pulse for the fast flux coil is generated by an AWG and is sent into the cryostat without passing a DC-block. Between the 70 K and 4 K-plate, the pulse is guided through a 20 dB attenuator and between the 100 mK and the 20 mK-plate it passes an Eccosorb filter and a DC-12 GHz bandpass filter. After filtering, the pulse is sent to the fast flux coil inside the hose.

The flux bias coils are connected directly to the constant current sources outside the cryostat without any attenuation or filtering. When entering the fridge the wires are wound extensively around a pole for better thermalisation. Then these wires are connected to the superconducting wires for the large bias coils between the 100 mK and the 20 mK-plate. The superconducting wire is necessary to have no resistance and thus no heating caused by the current.

Setup For Detection

The excitation of the qubit and the subsequent measurement of the cavity transmission are performed in two different setups. The first setup is used to perform a two tone scan, which is efficient in finding the qubits transition frequency. The second setup is used to generate pulses for a precise qubit control. The second setup can only be used when the qubit's transition frequency is known. The setup for detecting the cavity's transmission is the same in both qubit excitation schemes.



Figure 3.2: Connection to the cavity in the cryostat.



Figure 3.3: Setup for excitation and detection.

Two Tone Scan: Two microwave generators are used, referred to as EXG I and EXG II. One is used to excite the qubit inside the cavity by applying a saturation pulse. The other one is used to read out the qubit's state and therefore measures the transmission of the cavity. Both outputs of the microwave generators are combined in one power combiner. The output of the power combiner is connected to the cryostat input line for the cavity.

Pulse Scan: A microwave pulse of Gaussian shape cannot be generated directly by one of the microwave generators. Therefore a Gaussian pulse is generated by the AWG at a frequency of 100 MHz and is mixed up to the frequency of the EXG II using IQ-mixing. The output of the EXG II is used as local oscillator and is connected to port L of the IQ-mixer. The AWG signal is provided at two of its outputs. Both output signals are attenuated by a 20 dB attenuator and are connected to either the I or the Q port of the IQ-mixer. The mixed signal leaves the IQ-mixer at port R and enters an amplifier. A 3 dB attenuator between Port R and the amplifier lowers the signal such that, the amplifier works in its linear regime. The amplifier is connected to a directional coupler. The directional coupler sends a small part of its input to a spectrum analyser for calibration of the IQ-mixing. Finally the signal passes a bandpass filter at the range of the qubit's transition frequency and enters the power combiner.

Detection: The cavity output cannot be recorded directly by a digitiser, because the digitiser cannot handle signals above 50 MHz. Therefore the output signal is mixed down to 10 MHz before recording it. The output line for the cavity on top of the cryostat is connected to port R of the downmixer. Another signal generator, AnaPico, is used as local oscillator for the downmixer and is connected to Port L. The AnaPico provides a frequency that is 10 MHz above the EXG I frequency. The downmixed signal leaves Port I and enters a DC-80MHz filter to obtain the down mixed signal only. After filtering the signal is amplified by a factor of 125 and sent to the digitiser.

3.2.2 Qubit Characterisation Measurements

To achieve full control on the qubit, it has to be characterised first. The characterisation is done in well defined steps, that are introduced next.

First Qubit Test

First the cavity's resonance frequency is measured at high power. The resulting peak is referred to as high power peak. Next the power is decreased stepwise. At sufficiently low power, the resonance frequency jumps to a higher frequency if the qubit inside the cavity is working. The resulting peak is referred to as low power peak. In case of a SQUID based transmon one may apply a magnetic field. The cavity resonance frequency moves if the SQUID loop is working. The low power peak is used to perform a dispersive read-out of the qubit's state.

Figure 3.4 shows the power dependent resonance frequency shift of a cavity with a single qubit placed inside it. The frequency starts to shift above -24 dBm from the high power peak to the low power peak. The low power peak is measured for a power below -35 dBm. The shift between both peaks is 2,26(4) MHz on average.

Qubit Transition Frequency

Next the qubit's transition frequency ω_q is determined by performing a two tone scan. A saturation pulse is swept through the frequency range, where the qubit's transition frequency is expected. Immediately after each applied saturation pulse, the cavity's transition is measured at its low power peak. A wait time of one millisecond is necessary between every measurement sequence. The waiting is essential to be sure, that the qubit decays to its ground state before applying the next saturation pulse. If the qubit's transition frequency is hit, the read-out peak shifts to a lower frequency. As a result a lower transmission is measured, revealing the qubit's transition frequency.

Figure 3.5 shows the detection of a quantum bit at 3,89349(3) GHz. The transmitted signal drops about 1,5 mV due to the dispersive shift as the qubit is excited. The measured data points are fitted by a Lorentzian function, mainly to gain information about the resonance frequency. Additional information is in the width of the peak, which is related to the qubits lifetime and the power used for exciting the qubit. A higher excitation power results in a broader peak. It is helpful to use a relatively high excitation power to find the qubit's transition frequency in a first scan. The amplitude of the peak is a measure for the read-out contrast. The contrast can be optimised by varying the excitation power and read-out power.

Flux Tunability

The transition frequency of a SQUID based transmon is tunable by applying a magnetic field through the SQUID loop. Changing the transition frequency of the qubit varies the cavity's resonance frequency. The applied magnetic field is generated by the large bias coils. The cavity's resonance frequency is tracked for different current throug



Figure 3.4: Power dependent resonance frequency shift of the cavity. In the upper figure the cavity transmission is plotted for different input power of -20 dBm (blue), -25 dBm (red), -30 dBm (green), -40 dBm (purple) and -50 dBm (orange). The data points in the lower plot are gained from a frequency scan on the cavity for different powers, like in the upper figure. Each point in the plot results from a Lorentzian fit of the corresponding frequency scan, giving the central frequency of the measured peak.



Figure 3.5: Detection of a qubit. The transmitted signal at the cavity's resonance frequency drops as the qubit is excited. The data points are fit by a Lorentzian function (red line) to get the transition frequency.

the coils to find the flux sweet spots of the qubit. At a sweet spot the flux noise is minimised and the qubit works best.

Figure 3.6 shows the tunability of the cavity's resonance frequency by applying a current through the large bias coils. The data points are fitted by the function

$$f(I) = f_c - \frac{g^2}{f_c - f_q \sqrt{|\cos(\pi\omega(I - I_0)/\Phi_0)|\sqrt{1 + d^2 \tan(\pi\omega(I - I_0)/\Phi_0)^2}}}$$
(3.1)

that results from calculations done in [28]. One obtains a low frequency and a high frequency sweet spot for a current of -257(5) μ A and 329(11) μ A respectively. The periodicity $\frac{\Phi_0}{2\omega}$ is 1170(20) μ A. The other parameters like the coupling g = 0.8 GHz, the asymetry parameter d = 0.9, the central qubit transition frequency $f_q = 1.98$ GHz and the bare cavity resonance frequency $f_c = 9.36$ show an error, that is orders of magnitudes larger than the result from the fit.

In this case the measurement results are mainly helpful to estimate the sweet spots and the periodicity. The same measurement can be done with tracking the qubit frequency instead of the cavity frequency in principle. The qubit used in this experiment is only detectable in a small frequency range around its high frequency sweet spot. Therefore it is not possible to track the qubit along its full range. In further measurements the qubit is parked next to the high frequency sweet spot.



Figure 3.6: Flux tunability of the cavity frequency. The cavity frequency shifts by varying the current through one coil outside the cavity. A fit (red line) determines the required current for placing the qubit at one of its sweet spots.



Figure 3.7: High power excitation measurement. Exciting the qubit at high power reveals a transition to the second excited state next to the broad transition to the first excited state.
Anharmonicity α

The anharmonicity is measured by exciting the qubit at high power. Due to the high power an additional transition appears, exciting the qubit's next higher state. The additional transition is a two photon transition. Two photons at a slightly lower frequency excite the second level of the qubit with very low probability. If very high power is used, the two photon transition is more likely to happen and can be detected. The difference between the single and the two photon transition is used to gain information on the anharmonicity.

Figure 3.7 shows a very broad peak at $f_{01} = 3,931$ GHz, which is the transition frequency for the first excited qubit state at the sweet spot. Two peaks appear at $f_{02/2}^{(H)} = 3,782$ GHz and $f_{02/2}^{(L)} = 3,778$ GHz, describing a higher and lower two photon transition to the second excited qubit state respectively. In theory only one peak should appear for the two photon transition. The appearance of two peaks may be related to charge noise inside the qubit. To estimate the anharmonicity the average of $f_{02/2}^{(H)}$ and $f_{02/2}^{(L)}$ is taken first to get a mean two photon transition frequency $f_{02/2} = 3,780$ GHz. The anharmonicity α is given by [28]

$$\alpha/2\pi = 2(f_{02/2} - f_{01}) \tag{3.2}$$

and results in -300,9(3) MHz for this qubit at the upper flux sweet spot. The capacitive energy is related to the anharmonicity by the equation

$$\alpha = -E_c/\hbar \tag{3.3}$$

directly. Since ω_q and E_c is known from previous measurement, the Josephson energy E_J is calculated by

$$\hbar\omega_q = \sqrt{8E_C E_J} - E_C \tag{3.4}$$

and gives $E_J = 7,439(3)$ GHz. The ratio E_J/E_C is 24,72(3).



Figure 3.8: Rabi oscillation. The qubit's transition frequency is 3,925 GHz. A Gaussian pulse of 50 ns duration is used to excite the qubit at its transition frequency. The power of the excitation pulse is increased resulting in oscillations between ground and excited state of the qubit. The data points are fitted by a cosine function (red line) to get the periodicity.

Rabi Oscillation

As soon as the qubit's transition frequency is known, the setup is switched to the pulse measurement setup. The transition frequency is convoluted with a Gaussian pulse of different length and power. The Gaussian pulse should be as short as possible below 100 ns to achieve fast qubit control. Therefore the power of the pulse is increased stepwise for different pulse lengths. After each pulse the qubit state is measured. The excited state population is plotted against the power of the Gaussian pulse. One observes a power dependent Rabi oscillation. From that the required power for performing a π - and a $\pi/2$ -pulse is known at shortest pulse length.

Figure 3.8 shows a Rabi oscillation for a 50 ns short pulse at different power. The measurement is taken for a qubit transition frequency of 3,925 GHz, which is close to the upper flux sweet spot. The oscillation is fit with a cosine function to get the periodicity. From the periodicity the required power for a π -pulse and a $\pi/2$ -pulse follows. The periodicity is 0,81(5) V.



Figure 3.9: T_1 -measurement. The measurement shows an exponential decay of the excited state after excitation at $t_1 = 0$. The data points are fitted by an exponential function (red line) to determine the coherence time $T_1 = 8,0(4) \mu s$.

Coherence Time T_1

To measure the coherence time T_1 the qubit is excited by a π -pulse first.



After the excitation one waits for a time t_1 and then reads out the qubit's state by applying a read-out pulse (R0). One observes an exponential decay of the qubit for increasing the time between excitation and measurement. The coherence time T_1 is determined by fitting an exponential function to the measured data.

Figure 3.9 shows the result for a T_1 -measurement. The data points are fitted by an exponential function, revealing a coherence time of 8,0(4) μ s.

Dephasing Time T₂

The dephasing time determines the time it takes the qubit to lose its phase. The measurement sequence consist out of a $\pi/2$ -pulse, followed by a variable wait time t_2 and a second $\pi/2$ -pulse.





Figure 3.10: T_2 -measurement. The data shows an oscillation between the ground and the excited state, that saturates in a superposition of both states. An exponential fit multiplied with two cosine functions at different frequencies (red line) gives the dephasing time $T_2 = 1,04(9) \, \mu s$.

The qubit's state is read out after the total sequence. This is repeated for different wait times t_2 to observe an exponential decay of the qubit state. Fitting the measured data with an exponential decay times two cosine functions of different frequencies gives the dephasing time T_2 . The exponential function describes the decay, the two cosine functions describe two different transition frequencies due two charge noise.

Figure 3.10 shows the result of a T_2 -measurement. The qubit's transition frequency is at 3,925 GHz. The oscillation decays and is characterised by a dephasing time of 1,04(9) μ s resulting from the fit. At about 2 μ s the oscillation revives and is damped completely afterwards.

Improvement By T_{echo}

The determination of T_{echo} is similar to the measurement sequence of the dephasing time. An additional π -pulse is performed in the middle of the two $\pi/2$ -pulses.



The $\pi/2$ -pulses are separated by a variable time t_{echo} before the read-out is done. The measured data is fit to an exponential decay multiplied with a cosine to determine T_{echo} . Due to this measurement sequence the dephasing is reduced.



Figure 3.11: T_{echo} -measurement. The measurement shows a damped oscillation between the excited and the ground state. The decay time is given by an exponential fit multiplied with a cosine (red line) and gives $T_{echo} = 6,0(3) \mu s$

Figure 3.11 shows the data of a T_{echo} -measurement for the qubit at 3,925 GHz. The exponential decay of the oscillation is 6,0(3) μ s.

Issues During Characterisation

The qubit transition frequency is very low and misses the designed average value of 6,9 GHz by more than 3 GHz. Therefore the detuning between cavity and qubit is very large resulting in a low dispersive shift. By tuning the qubit to lower frequencies this effect enhances until the qubit is not detectable anymore. In this measurements the qubit can be tracked between 3,93 and 3,75 GHz. Below 3,75 GHz the qubit peak is split sometimes or vanishes completely. For this reason further measurements can only be done within this relatively small range.



Figure 3.12: Square pulse and square pulse. In a) a normalised square pulse is plotted. If the square pulse is extended periodically, it becomes a square wave, depicted in b). A square wave is approximated by a finite Fourier series for n = 10 in c) and n = 100 in d).

3.3 Fast Flux Pulse Measurements

Before measurements with fast flux pulses are presented, square pulses are discusses shortly in theory. Next the realisation of a square current pulse is investigated experimentally with an AWG. The finite sample rate of the AWG limits the sharpness of the pulses. Current pulses of a rise time below 100 ns are sent to the fast flux coil inside the hose for the final test. The pulse are characterised in the final measurments.

3.3.1 A Square Pulse

A square pulse starts with an instantaneous change from no signal to a fixed amplitude. The amplitude is held for a finite time until it falls back immediately to no signal. This is shown in figure 3.12a. One can extend a square pulse periodically and calculate the Fourier series

$$f_{\omega}(t) = \frac{4}{\pi} \sum_{0}^{\infty} \frac{\sin((2k+1)\omega t)}{2k+1}$$
(3.5)

describing a normalised square wave of frequency ω , depicted in figure 3.12b. The square wave consists out of an infinite series of sinusoidal functions with growing frequency and decreasing weighting. Since a square pulse is equivalent to a square wave considering only half a period, it has the same infinite frequency components. The figures 3.12c and 3.12d show a square wave that is approximated by a finite Fourier series.

Imagine a square pulse is realised by a current I(t) applied to a coil. The coil

transfers the current into a magnetic pulse B(t), ideally of the same shape, to control a SQUID based qubit. However, the magnetic pulse may get deformed on the way to the qubit, because of two main reasons. First, the circuit including the coil may act as a filter. Second, there are conducting surfaces next to the coil, that may attenuate the magnetic pulse due to eddy currents. Both situations result in attenuation and phase shifting of some frequency components of the applied pulse. To solve this problem one uses concepts from signal processing theory for linear and time invariant systems.

3.3.2 Pulse Generation

The current through the small coil in the hose is provided by an arbitrary wave generator (AWG). The AWG accepts a programmed signal in the range of -1,5 to 1,5 V at a sample rate of 200 MS/s. Therefore every second nanosecond the voltage can be set to an arbitrary value within the given range. The AWG tries to realise the output signal according to the programmed input. But this comes along with some troubles, when realising a square pulse.

Due to the finite sample rate a perfect square pulse cannot be generated. This fundamental problem is known as Gibbs phenomenon. Frequency components higher than 0,5 GHz cannot be provided by the AWG. These high frequencies are missing in the pulse, leading to ripples after the rise and fall. Gibbs ringing thus overshoots the expected pulse amplitude by approximately 9 % maximally.

Additionally to Gibbs ringing, the AWG overshoots the pulse at the beginning to handle the fast rise and undershoots it at the end to handle the fast fall. The AWG output for an initial programmed square pulse is shown in figure 3.13. The overshoot and undershoot are about 28 % more than the expected pulse amplitude and are followed by some ringing for 25 ns approximately. The amplitude of this ringing is more than one would expect from the Gibbs phenomenon only.

The strong ripples are a problem for the precise qubit control and should be avoided. The first try to get rid of the ringing is to use signal processing theory as explained in appendix B. But this approach does not work for the AWG. One reason is the fundamental limit of the Gibbs phenomenon. Another one is that the AWG is not an LTI system as some amplifiers or in general non-linear elements are part of it. These elements might not be able to handle the fast change in signal and result in additional overshooting.

A suitable solution is offered by replacing the sharp steps of the square pulse by a smooth transition as short as possible. The shortest transition time without any ringing is achieved so far by using the normalised function

$$p(t) = \begin{cases} \left| \sin\left(\frac{\pi}{2\tau_r}t\right) \right| \\ 1 \\ 1 - \left| \sin\left(\frac{\pi}{2\tau_r}\left(t - (\tau_r + T)\right) \right) \right| \end{cases} \quad \text{for } t \in \begin{cases} [0, \tau_r) \\ [\tau_r, \tau_r + T] \\ (\tau_r + T, 2\tau_r + T] \end{cases}$$
(3.6)



Figure 3.13: Rise and fall of different square pulses generated by the AWG. The output of the AWG is measured for different rise times τ_r of 0 ns (blue), 4 ns (red), 8 ns (green), 12 ns (orange), 16 ns (purple) and 20 ns (pink). The additional rise time τ_r is added twice to the pulse of length $T = 1 \,\mu$ s, once at the beginning and once at the end.

as pulse of length *T* with smooth rise time $\tau_r \ge 16$ ns for programming the AWG. The resulting pulse is measured for different rise time and is plotted in figure 3.13.

3.3.3 First Fast Flux Pulse Measurements

This subsection presents the first fast flux pulse measurements. There the idea is to apply a fast flux pulse (FFP) during a T_1 -measurement.



A fast flux pulse is applied at a fixed time for a duration of some microseconds after exciting the qubit. Optimally the qubit coherence time is not affected by the flux pulse. Still an effect of the fast flux pulse is expected to be visible, since the cavity's resonance frequency and thus the read-out peak is shifted due to the flux pulse.

Before starting the measurement sequence the qubit is set to one of its high frequency sweet spots by one of the two large bias coils. Then the measurement sequence is started leading to the results depicted in figure 3.14. A flux pulse is applied after $3 \mu s$ for a duration of $4 \mu s$ in the upper and after $1 \mu s$ for a duration of $2\,\mu s$ in the lower figure 3.14. Both flux pulses are generated by an AWG. An instant current (like the blue pulse in figure 3.13) is sent from the AWG to the fast flux coil inside the hose. The flux pulse for the measurement in the upper figure 3.14 is generated by a current of 5,7 mA. Double the current is used for the flux pulse in the measurement depicted in the lower figure 3.14. A data point is taken every 100 ns in both measurements. One can clearly see the effect of the flux pulse on the read-out peak additionally to the exponential qubit decay. The read-out peak is shifted towards the read-out frequency. The shift becomes stronger for stronger flux pulses. After the pulse ends, the bare qubit decay is observed again. This indicates, that the applied flux pulses have an effect on the qubits transition frequency but keep the qubit state partially excited. The rise time of the flux pulse is roughly estimated below 200 ns in the upper figure 3.14 and below 500 ns in the lower figure 3.14.

From the above measurements no information is gained about the shifted qubit frequency. Therefore another measurement is taken, that tracks the qubit frequency. A qubit spectroscopy is done during a fast flux pulse in discrete time steps.



The fast flux pulse (FFP) is swept through the spectroscopy scheme consisting out of an excitation pulse (EXP) and read-out pulse (RO). The result is a shown in figure 3.15.

The qubit is parked at a transition frequency of about 3,81 GHz by one of the two large bias coils. A fast flux pulse is generated for a duration of 20 μ sby the AWG (like



Figure 3.14: Fast flux pulse during a T_1 -measurement. The flux pulse starts $3 \mu s$ (1 μs) after the initial excitation of the qubit for a duration of 4 μs (2 μs) in the upper (lower) measurment. A current pulse of 5,7 mA (11,4 mA) is sent through the small coil in the hose to generate the flux pulse.



Figure 3.15: Two tone scan of a fast flux pulse induced frequency shift. From blue to red the read-out signal increases.

the blue pulse in figure 3.13). The provided current is 2,9 mA. The qubit is shifted about 50 MHz away from an initial frequency after 1 μ s and jumps back as the pulse ends. One can fit the rise of the pulse with an exponential function. The resulting rise time is 0,8(2) μ s.

The qubit does not exactly jump back to its initial frequency, the frequency increases slightly along the whole scan. A possible reason for this effect might be a too short wait time of $1 \mu s$ between two single scans. If the wait time is too short, residual magnetic field might be inside the cavity and influence the measurement of the next scan. The read-out scheme stays constant in time and the magnetic pulse is swept towards the end of the sequence for each scan. Therefore it might have a bigger influence on the shift at the end of the whole scan. Another possibility explaining the shift is that the hose might be magnetised over time by applying a pulse in single direction.

The measurements so far are done without making use of the smooth AWG pulse output shown in section 3.13. As a consequence the overshooting current at the edges of the pulse might have a bad effect on the fast flux coil and decrease the rise time of the magnetic field. Further measurements in the next section make use of a smooth pulse.

3.3.4 Further Fast Flux Pulse Measurements

In the following fast flux pulse measurements a slightly different setup is used. Two SQUID based transmon qubits are placed inside the cavity, one in the centre of the cavity and one next to it as depicted in figure 3.1. Additional some sapphire is put into the qubit slots at the edge of the cavity. The additional sapphire decreases the cavity frequency and thus the detuning between cavity and qubit. As a result the dispersive shift is increased, which improves the read-out.

The idea of putting two qubits inside the cavity is to let them interact with each other. For an unknown reason it is not possible to detect the single qubits during a spectroscopy measurement with these qubits in this setup. Only by accident an unknown excitation is detected. The excitation is achieved by applying the excitation and the read-out pulse of a two tone scan at the same time. Applying both pulses at the same time is unwanted, because the qubit might get excited to a higher unknown state. Luckily this unknown excitation is flux tunable and further investigations on the fast flux pulse can be done. The excitation is detectable between 3,7 and 3,5 GHz, where 3,7 GHz is the high frequency sweet spot.

First the behaviour of a long flux pulse is investigated. The applied smooth flux pulse has a rise time of 100 ns. A two tone scan is done during a flux pulse is applied as depicted in figure 3.15. This time a wait time of 1 ms between each scan is introduced to compensate the shift due to residual fields from a previous scan. The minimum of the whole scan is extracted to get a line plot.

Figure 3.16 shows the extracted minimum of a measurement, where a flux pulse is applied for a duration of $100 \,\mu$ s. After the sharp rise at $10 \,\mu$ s the frequency is expected to be constant. Here an increase of about 10 MHz is measured. The increase of the frequency is related to an increase of magnetic field in the cavity. The field inside the cavity can be increased due to two possibilities, one is connected to eddy currents and one to the magnetisation of the hose. The instant rise of the magnetic field causes eddy currents in the cavity walls counteracting the change in magnetic flux through it. As the applied field turns constant, the flux through the cavity walls is not changed anymore and the counteracting eddy currents decay. Another cause might be a possible magnetisation of the hose. The constant magnetic field magnetises the hose. As the field is turned off, the magnetisation might decay slowly. Both effects increase the magnetic field in the cavity with time and shift the frequency slightly.

The same effect is observed when the flux pulse is switched of as depicted in figure 3.17. There the pulse is switched of at 1 μ s. The qubit's transition frequency jumps as expected, but does not stay constant. Again a slow drift is observed that is related to eddy currents resulting from the fast change of flux through the cavity walls. The drift in the opposite direction towards a lower frequencies does not depend on whether the flux pulse is switched on or off. It depends whether the excitation is detuned towards or away from its sweet spot. In figure 3.17 the excitation is detuned above the sweet spot.

Finally a double flux pulse is applied during a spectroscopy scan. The rise time



Figure 3.16: Scan of a long fast flux pulse towards the high frequency sweet spot.



Figure 3.17: Scan after a fast flux pulse is switched off.



Figure 3.18: Scan of a fast flux double pulse.

of the double flux pulse is set to 20 ns whenever the constant field changes. Figure 3.18 shows the measurement results. An initial flux pulse is applied at $10 \,\mu s$ for a duration of $20 \,\mu s$ using a current of 2,86 mA. Then an inverted pulse follows for another $20 \,\mu s$ using the same peak current. The region at $30 \,\mu s$, between the first and the second inverted pulse, is scanned in 200 ns steps. The jump from 3,708 to 3,588 GHz happens between two data points and thus it is faster than 200 ns. For this jump a total current of 5,71 mA through the fast flux coil is applied.

The sequence starts and ends at the same frequency when no magnetic field is applied. This indicates that the hose is not magnetised at the end of this sequence and shows no hysteresis. Otherwise the frequency at the end would be shifted by a constant value with respect to the initial frequency. Before the jump at $30 \,\mu s$ starts, an overshoot is detected. The AWG does not cause the overshoot, because the output of the sequence is measured with an oscilloscope before attaching the AWG to the fast flux coil. Like in the measurements before, the constant part of each pulse shifts slightly due to eddy currents or a magnetisation effect. The second pulse is shifted stronger than the first one, because it is farther away from the high frequency sweet spot.

3.4 Summary

A magnetic hose can be used to send a magnetic pulse from the outside to the inside of a microwave cavity such that, the transition frequency of a SQUID based transmon qubit is controllable by the applied magnetic flux pulse. The characterisation of the fast flux pulses show that a switching between 3,708 to 3,588 GHz below 200 ns is possible sending a current of 5,71 mA through the fast flux coil. Since the AWG is able to handle currents of 42 mA_{PP} it should be possible to switch between any possible transition frequency within this timescale. It is not possible to show full flux tunability on this qubit, because it is undetectable at low frequencies.

A flux pulse shows a slight drift during its constant regime. It is supposed to be related to a counteracting effect that decays over time. Such a counteracting effect might be provided by eddy currents inside the cavity or magnetisation of the hose. This effect, however, is on slow timescales and can be fixed by initially applying a correction pulse, that takes the slight drift into account. If the drift depends mainly on eddy currents it might be reduced by putting a second hose inside the cavity from the opposite direction.

As shown in figure 3.14 the qubit stays partially in the excited state when a fast flux pulse is applied. A similar measurement can be done for the dephasing time. There the fast flux pulse should be visible by a phase jump of the oscillation. Since the dephasing time of the used qubit is about $1 \mu s$, the measurement will not lead to a result.

All measurements are done with a relatively bad qubit. It is supposed that, using a better qubit will lead to clearer results and it will be able to test the tunability in full range. Additionally the characterisation of the fast flux pulses will be more precise and flexible. A clear tracking of the qubit will make characterisation of the fast flux coil feasible, which is not possible with this qubit. Despite the relatively bad qubit the experiments show a clear result. Fast flux control on a transmon qubit in a three-dimensional cavity architecture is possible using a magnetic hose, that guides a magnetic pulse from the outside of the cavity to the inside.

Outlook

The result of this thesis is clear. A magnetic hose enables the transport of a fast flux pulse into a microwave cavity to control the transition frequency of a quantum bit. In the experiments so far a cavity made out of copper is used. The presented technique is assumed to permit fast flux control even in superconducting cavities made out of aluminium.

The main idea of guiding magnetic fields is always the same. Superconducting and ferromagnetic elements have to be combined in the right way. A smart combination of the required materials might improve the performance of the hose and change the design such that thinner hoses can be built. Guiding and shaping magnetic fields might find more applications than in a magnetic cloak [37], a magnetic hose [29] or a magnetic wormhole [38].

Fast flux control on a quantum bit is a step closer to realise quantum simulations using superconducting qubits and might lead one day to a universal quantum computer. The next step will be to test the setup with one flux sensitive and one non flux sensitive quantum bit. There the idea is to investigate the interaction between these two quantum bits. The flux sensitive quantum bit can be tuned and thus both quantum bits can be excited and detected individually. This scheme will be extended to more quantum bits in arbitrary geometries to simulate spin chains [39] in the next five years.

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Appendix A

Dispersive Hamiltonian

The Hamiltonian (1.9) is split into two parts $H_{ac} = H_0 + H_1$, where:

$$H_0 = \hbar \omega_c a^{\dagger} a + \frac{\hbar \omega_a}{2} \sigma_z \tag{A.1}$$

$$H_1 = \hbar g(\sigma_+ + \sigma_-)(a + a^{\dagger}) \tag{A.2}$$

The unitary transformation $U = e^{-i\frac{H_0}{\hbar}t}$ transforms H_{ac} into the interaction picture via

$$H_I = i\hbar \dot{U}^{\dagger} U + U^{\dagger} H_{ac} U \tag{A.3}$$

resulting from the Schrödinger equation. Since $[U, H_0] = 0$, this gives:

$$H_{I} = \hbar g \left(e^{i(\omega_{c}a^{\dagger}a + \frac{\omega_{a}}{2}\sigma_{z})t} (\sigma_{+} + \sigma_{-})(a + a^{\dagger}) e^{-i(\omega_{c}a^{\dagger}a + \frac{\omega_{a}}{2}\sigma_{z})t} \right)$$
(A.4)

One can show that:

$$\sigma_z \sigma_+ = \sigma_+ \qquad \qquad \sigma_z \sigma_- = -\sigma_-$$

$$\sigma_+ \sigma_z = -\sigma_+ \qquad \qquad \sigma_- \sigma_z = \sigma_-$$

is valid and use $[a, a^{\dagger}] = 1$ to simplify

$$e^{i\omega_{c}a^{\dagger}at}ae^{i\omega_{c}a^{\dagger}at} = ae^{-i\omega_{c}t} \qquad e^{i\frac{\omega_{a}}{2}\sigma_{z}t}\sigma_{+}e^{-i\frac{\omega_{a}}{2}\sigma_{z}t} = \sigma_{+}e^{i\omega_{a}t}$$
$$e^{i\omega_{c}a^{\dagger}at}a^{\dagger}e^{i\omega_{c}a^{\dagger}at} = a^{\dagger}e^{i\omega_{c}t} \qquad e^{i\frac{\omega_{a}}{2}\sigma_{z}t}\sigma_{-}e^{-i\frac{\omega_{a}}{2}\sigma_{z}t} = \sigma_{-}e^{-i\omega_{a}t}$$

by rewriting the exponential functions into their series representations. Using these relations gives:

$$H_{I} = \hbar g \left(\sigma_{+} a e^{i(\omega_{a} - \omega_{c})t} + \sigma_{-} a^{\dagger} e^{-i(\omega_{a} - \omega_{c})} \right) + \hbar g \left(\sigma_{+} a^{\dagger} e^{i(\omega_{a} + \omega_{c})t} + \sigma_{-} a e^{-i(\omega_{a} + \omega_{c})t} \right)$$
(A.5)

For $\Delta = \omega_c - \omega_a = 0$ one obtains:

$$H_{I} = \hbar g \left(\sigma_{+} a + \sigma_{-} a^{\dagger} + \sigma_{+} a^{\dagger} e^{i2\omega_{a}t} + \sigma_{-} a e^{-i2\omega_{a}t} \right)$$
(A.6)

The last two terms are neglected due to the rotating wave approximation. Therefore the Hamiltonian in the interaction picture is

$$H_I = \hbar g (\sigma_+ a + \sigma_- a^{\dagger}) \tag{A.7}$$

and the full Hamiltonian gives the Jaynes-Cummings Hamiltonian:

$$H_{JC} = \hbar\omega_c a^{\dagger}a + \frac{\hbar\omega_a}{2}\sigma_z + \hbar g(\sigma_a a^{\dagger} + \sigma_+ a)$$
(A.8)

From first order perturbation theory follows, that there are no energy shifts $E_n^{(1)} = \langle n^{(0)} | V | n^{(0)} \rangle$ in first order approximation. Thus second order terms have to be considered next. Girvin presents a way to calculate the corresponding Hamiltonian in his lecture notes [40] starting at page 77. The Hamiltonian $\tilde{H}_{JC} = UH_{JC}U^{\dagger}$ is calculated by assuming a unitary transformation $U = e^{e\hat{t}a}$ that deletes all first order terms. Using the Baker-Campbell-Hausdorff formula to second order gives:

$$\tilde{H}_{JC} \approx H_0 + H_I + [\hat{\eta}, H_0] + [\hat{\eta}, H_I] + \frac{1}{2} [\hat{\eta}, [\hat{\eta}, H_0]] + \frac{1}{2} [\hat{\eta}, [\hat{\eta}, H_I]]$$
(A.9)

The lowest order off-diagonal term has to be 0. Therefore $\hat{\eta}$ has to satisfy $[\hat{\eta}, H_0] = -H_I$, which corresponds to:

$$\hat{\eta} = \frac{g}{\Delta} \left(a\sigma_+ - a^{\dagger}\sigma_- \right) \tag{A.10}$$

Therefore the approximated Hamiltonian is:

$$\tilde{H}_{JC} = H_0 - \frac{1}{2} [\hat{\eta}, [\hat{\eta}, H_0]]$$
(A.11)

Next the commutator

$$[\eta, H_0] = \frac{g}{\Delta} (a\sigma_+ - a^{\dagger}\sigma_-)(\hbar\omega_c a^{\dagger}a + \frac{\hbar\omega_a}{2}\sigma_z) - (\hbar\omega_c a^{\dagger}a + \frac{\hbar\omega_a}{2}\sigma_z)\frac{g}{\Delta}(a\sigma_+ - a^{\dagger}\sigma_-) \quad (A.12)$$

is calculated, giving:

$$[\eta, H_0] = \frac{\hbar g}{\Delta} \left(\omega_c (a\sigma_+ + a^{\dagger}\sigma_-) - \omega_a (a\sigma_+ + a^{\dagger}\sigma_-) \right) =$$
(A.13)

$$= -\hbar g_q (a\sigma_+ + a^{\dagger}\sigma_-) \tag{A.14}$$

The expression

$$[\eta, [\eta, H_0]] = \frac{g}{\Delta} (a\sigma_+ - a^{\dagger}\sigma_-) \left(-\hbar g (a\sigma_+ + a^{\dagger}\sigma_-) \right) - \left(-\hbar g (a\sigma_+ + a^{\dagger}\sigma_-) \right) \frac{g}{\Delta} (a\sigma_+ - a^{\dagger}\sigma_-) \quad (A.15)$$

can be simplified using the relations from above and:

$$\begin{array}{ll} \sigma_{+}\sigma_{+}=0 & \sigma_{+}\sigma_{-}=\left|1\right\rangle\left\langle 1\right| \\ \sigma_{-}\sigma_{-}=0 & \sigma_{-}\sigma_{+}=\left|0\right\rangle\left\langle 0\right| \end{array}$$

This gives:

$$\begin{split} \left[\eta, \left[\eta, H_{0}\right]\right] &= -\frac{\hbar g^{2}}{\Delta} \left(2a^{\dagger}a\sigma_{+}\sigma_{-} - 2a^{\dagger}a\sigma_{-}\sigma_{+} + 2\sigma_{+}\sigma_{-}\right) = \\ &= -\frac{\hbar g^{2}}{\Delta} \left(2a^{\dagger}a\sigma_{+}\sigma_{-} - 2a^{\dagger}a\sigma_{-}\sigma_{+} + \sigma_{+}\sigma_{-} - \sigma_{-}\sigma_{+} + 1\right) = \\ &= -\frac{2\hbar g^{2}}{\Delta} \left(a^{\dagger}a\sigma_{z} + \frac{1}{2}\sigma_{z} + \frac{1}{2}1\right) \end{split}$$
(A.16)

The relative energy shift is dropped and it follows:

$$[\eta, [\eta, H_0]] = \frac{-2\hbar g^2}{\Delta} \left(a^{\dagger} a + \frac{1}{2} \right) \sigma_z \tag{A.17}$$

The result is put into equation (A.11) to give the Hamiltonian in the dispersive limit:

$$H_{JC} = \hbar \omega_c a^{\dagger} a + \frac{\hbar \omega_a}{2} \sigma_z + \frac{\hbar g^2}{\Delta} \left(a^{\dagger} a + \frac{1}{2} \right) \sigma_z$$

Appendix B

Signal Processing Theory Considering LTI Systems

An initial signal I(t) is sent and underlies the unknown influence of a linear and time invariant (LTI) system. The received signal R(t) might be deviating from the initial signal. In other words, the system transfers the initial signal I(t) to an received signal R(t). For this reason, the system is characterised by an LTI transfer function H(t) that is unknown. Under this assumption the received signal is given by

$$R(t) = I(t) * H(t) = \int_{-\infty}^{\infty} I(\tau)H(t-\tau)d\tau \quad , \tag{B.1}$$

which is the convolution of I(t) and H(t).

One can see that the received signal would correspond to the initial signal, if a corrected signal

$$I_{corr}(t) = I(t) * H^{-1}(t)$$
(B.2)

is applied. Obviously this is true since $H^{-1}(t) * H(t) = \delta(t)$, where $\delta(t)$ is the Dirac delta function. As a consequence any effect of the system can be counteracted, if the system's transfer function or its inverse is known.

The system's transfer function can be directly measured, if $I(t) = \delta(t)$. In this case R(t) = H(t). Since a delta function is hard to realise experimentally, one has to find a different solution. Usually $I(t) = \Theta(t)$ is used, meaning the input signal equals the Heaviside function. The Heaviside function is realised by instantly switching on a constant signal. As a result the output signal equals

$$R(t) = \Theta(t) * H(t) = \int_{0}^{t} H(\tau) d\tau \quad , \tag{B.3}$$

which is the integral of the system's transfer function. The derivative of R(t) then gives the system's transfer function. To compensate the effect of the system one calculates the convolution of the signal of choice and the system's inverse transfer function to find the corrected signal.

A more general and efficient way is to conclude the system's transfer function from solving the convolution integral in Fourier or Laplace space. The convolution of two functions in time space corresponds to a multiplication in Fourier space:

$$I(t) * H(t) \simeq I(i\omega)H(i\omega)$$
(B.4)

Therefore

$$H(iw) = \frac{R(i\omega)}{I(i\omega)}$$
(B.5)

for an arbitrary initial signal.

From a numerical point of view, it is easy to compute the Fourier transform with the FFT-algorithm. Solving the convolution numerically costs more resources and therefore takes more time. Thus it is preferable to calculate the system's inverse transfer function in Fourier space and bring it back to time space with the inverse Fourier transform.

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